

ROAD RESEARCH LABORATORY

Ministry of Transport

RRL REPORT LR 254

**TYRE FORCES AS FUNCTIONS OF CORNERING AND
BRAKING SLIP ON WET ROAD SURFACES**

by

K. E. Holmes and R. D. Stone B.Sc.

**Road Research Laboratory
Crowthorne, Berkshire
1969**

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ABSTRACT

The main purpose of this report is to present and discuss empirical data on braking and sideways forces both separate and combined, and on self aligning torque, all as functions of braking slip and slip angle. The data refers to wet road surfaces only, but the effect of such factors as speed, road surface texture, tread pattern, tread resilience, tyre construction and tyre wear are reported. The test vehicle recording system, data evaluation and analysis are also described.

It is suggested that a single relationship between the resultant force and the resultant slip, particularly if the true slip could be ascertained, might account for the larger part of the variations actually observed in the whole range of conditions from pure braking force through a combination of braking and sideways forces to pure sideways force. The importance of combined braking and cornering to the performance of antilocking brakes is shown. A theoretical study of the stopping distances implied by the braking force data under a variety of braking conditions, and statistical analyses of the pure braking and pure sideways force data are also included.

I. INTRODUCTION

The motion and the controllability of a vehicle depend mainly on the forces generated at the tyre-ground contacts and on how they vary under different dynamic conditions¹. These forces, spread over the contact area, are normally resolved into three force and three couple components at the centre of the contact area. Data on the force components and one couple, the self aligning torque, are presented here. Although these components are friction like in character, they vary with a great number of factors², so that it is a very lengthy process to study them or to present the results in their full complexity.

Braking force (or driving force) for a given tyre, speed, road condition etc. and when acting alone, varies with the amount of braking slip³. The functioning of brakes, the locking of wheels and the performance of antilocking devices are intimately connected with the form of the relation. Similarly, cornering force depends on slip angle, and steering and stability depend mainly on the form of the relationship. These are the most basic relationships. In general however, and especially under emergency conditions braking (or traction) and cornering forces are present together, and the basic

relationship of either force is modified by the presence of the other. Besides this, braking or driving forces make their own direct contribution to vehicle stability.

The data presented here is therefore mainly the basic relationships for braking force and cornering force, their modification by mutual action, and such inter-relations of the combined forces as appear to be of interest from the vehicle or tyre point of view or as throwing light on the interaction itself. A little data on self aligning torque under pure cornering force is also given.

The presentation of the data is mainly graphical, in which form the amount which can be given is very limited. In most cases additional information can be made available. The effects of various factors have been investigated in one or two cases by statistical methods (multiple regression) and although the formulae obtained are not yet very satisfactory in that they do not account for enough of the variability, the method is capable of further development and the results already obtained are given. Another manner of condensing the basic data is to examine the effect which the various factors have on some relevant features of vehicle behaviour. As an example of this the theoretical braking distance to standstill, calculated from curves of basic data and the results of statistical analysis of their dependence on speed, are presented to illustrate the effects of tread pattern, resilience and road surface. The data are also of direct interest from the point of view of the dynamics of a tyred wheel under various types of brake force application.

Descriptions of the test vehicle, the recording system, methods of testing, and the method of evaluating the data and correcting for certain inherent cross connections of the forces under combined operation are also given.

1.1 Notation

L	vertical load
B	braking force
S	sideway force
R	resultant force
BFC	$= B/L =$ braking force coefficient
SFC	$= S/L =$ sideway force coefficient
RFC	$= (B^2 + S^2)^{1/2}/L =$ resultant force coefficient
SAT	self aligning torque
V	vehicle speed
Q	braking slip
α	cornering slip angle
Re	tyre tread resilience
P	tyre tread pattern (takes value 0 or 1 only)
a_0	a constant term.

2. DETAILS OF TESTS

2.1 Test vehicle and procedure

The test vehicle is shown in Plate 1 and a description has been published⁴. Force measurements are made on a fifth wheel which can be set at a slip angle from 0 to 350 mrad (0–20°), and

whose angular velocity can be held at any desired value quite independently of the vehicle speed by means of a hydrostatic transmission from the normal vehicle drive.

The test wheel is mounted at zero camber in a parallel link system, Plate 2, and the load is not affected by braking force, except in the conditions explained below. The king pin has zero caster and inclination angles and terminates in two ball joints. The force measuring system is a strain gauged cruciform member mounted around the upper ball joint of the king pin. This transmits to a 12 channel galvanometer signals representing the three force components transmitted through this ball joint. Self aligning torque is measured by strain gauges on the steering arm.

The system was designed to measure load, cornering force, braking force and self aligning torque, the two other moments being ignored. Because of the position of the steering arm there is no cross coupling between the force in it and those measured at the upper ball joint. Tyre overturning moment appears in the measurement system as a side force, but is eliminated from the measurements by the calibration procedure in so far as static (calibration) conditions are the same as dynamic (test) conditions. The moment of the ground forces about the wheel spin axis (this includes the tyre rolling moment) is transmitted to the driving shaft and is not measured. When braking force alone, or sideways force alone, is acting the cruciform system at the upper ball joint measures load and braking force (or sideways force) with little cross coupling. When braking force and slip angle are applied simultaneously however, the driving shaft, which is no longer straight, transmits forces and couples to the wheel system which alter the load and also appear as a spurious side force in the measuring system. These are evaluated and, together with the cross couplings among the forces, eliminated from the results as described in the evaluation of results and in Appendix 1.

Magnetic pulse generators fitted to the test wheel and nearside front wheel of the vehicle provide the galvanometer records from which speed and slip are evaluated. Photo-electric pulse generators on the same wheels feed a visual digital slip meter which gives the operator a measure of the slip every 4m (12 ft).

All tests were made on wetted surfaces, the depth of water being of the order of 0.5 to 0.75mm (0.020 - 0.030 in). At the beginning of a series of tests a record of wheel rotation was obtained with the test wheel running freely in order to obtain its rolling radius for evaluation of braking slip. Before a test run the test wheel was raised clear of the ground to record force and moment zeros. The wheel was then lowered and the load applied as the vehicle accelerated to the test speed. The required amount of slip or slip angle was set about $1\frac{1}{2}$ seconds, and the recorder energised about $\frac{1}{2}$ second, before passing over the test section, which was about 9m (30 ft) long. In comparison with the state of affairs when a wheel is ordinarily braked and locked, the test may be regarded as steady state. If possible each test length included the junction between two adjacent types of surface, so that two surfaces were tested in each run. The film record was analysed as described later.

2.2 Test surfaces

Five of the road surfaces available at the Laboratory track have been used in the tests described here. They were chosen as giving a wide range of skidding resistance and surface texture, rather than as typical road surfaces. They are illustrated in Plates 3 to 7 and described in Table 1.

TABLE 1
Details of Test Surfaces

No.	Description	Texture	Plate No.
1	9.5 mm quartzite macadam carpet	rough, harsh	3
2	Fine cold asphalt	smooth, harsh	4
3	9.5 mm mixed aggregate macadam carpet*	rough	5
4	9.5 mm Bridport macadam carpet	rough, polished	6
5	Mastic asphalt	smooth, polished	7

* Aggregate is a mixture of those in 1 and 4

TABLE 2
Details of Test Tyres

No.	Make	Tread Pattern	Construction	Size	Resilience*	Hardness†
1	Dunlop	Smooth ≠	Crossed ply	5.00x16	55	63
2	Dunlop Gold Seal	Patterned	Crossed ply	5.25x16	51	58
3	Dunlop	Smooth ≠	Crossed ply	5.00x16	38	63
4	Dunlop Gold Seal	Patterned	Crossed ply	5.25x16	31	62
5	Dunlop C41	Patterned	Crossed ply	5.90x15	31	63
6	Pirelli Cinturato	Patterned	Textile banded radial ply	165x15	36	65
7	Dunlop SP41	Patterned	Textile banded radial ply	165x15	35	60
8	Michelin 'X Stop'	Patterned	Metal banded radial ply	165x15	39	60
9	Michelin 'X Rib'	Patterned	Metal banded radial ply	5.50x16	—	—
10	Michelin 'X Rib'	Worn ≠	Metal banded radial ply	5.50x16	—	—

* Percentage rebound measured on a modified B.S. Lupke pendulum at 20°C

† Measured at 20°C on Dunlop Scale

≠ Defined in Details of Tests – Test tyres 2.3

2.3 Test tyres

For convenience of reference the details of the tyres used are listed in Table 2. A 'smooth' tyre is a new tyre having a tread rubber depth equal to that of a new patterned tyre, and a 'worn' tyre is one in which the tread has been worn or buffed away until no tread pattern remains.

3. SUMMARY OF DATA PRESENTED IN THIS REPORT

The data presented in this Report have been drawn from a number of investigations. The six investigations concerned are listed in Table 3, from which can be seen the particular feature being investigated, the range of the various factors involved, and the various Tables and Figures where selected portions of the data are displayed. The complete data of the investigations though not yet published is available in most cases.

4. EVALUATION OF RESULTS

4.1 Method of Evaluation

The evaluation procedure was programmed for a Pegasus digital computer, as outlined below for the general case of combined sideways and braking forces. Evaluation for pure braking or sideways force was a simplified version of the general case.

Film records of the tests and calibrations were first analysed on a trace reader which produced a punched tape suitable for input to the program. Calibrations were carried out by applying known forces to the contact patch. For a given tyre calibration records provided information on the relationship between the units used on the trace reader and the physical units, and enabled calculation to be made of the cross couplings between primary forces. It was noted during calibrations that, owing to variation of the attitude of the suspension links with different tyres, the application of sideforce caused the weighbridge reading (vertical load) to change by a few per cent of sideforce; to correct for this an empirical factor was used in the program.

The first step in the evaluation was to convert the arbitrary trace reader units to physical units so giving the forces in newtons, slip angle in radians, self aligning torque in newton-metres, and so on. An iterative technique was used to correct the forces for mutual cross couplings, then followed corrections for the effect of brake torque on sideways force and load, as explained in Appendix 1. The direction of the resultant horizontal force was obtained together with a theoretical value of the resultant sliding speed of the contact patch and its direction. Definitions and formulae for braking slip and sliding speed are given in Appendix 1. The output from the computer consisted of a set of values on punched tape for each test reading: vertical load, braking force, sideways force, resultant horizontal force, braking force coefficient (BFC), sideways force coefficient (SFC), resultant horizontal force coefficient (RFC), direction of resultant horizontal force, test vehicle speed, percentage braking slip, cornering slip angle, self aligning torque, theoretical sliding speed of contact patch and its direction. This punched tape output is suitable for re-input and subsequent computer

TABLE 3
Summary of Test Conditions

Data Group No.	1	2	3	4	5	6
Quantity investigated*	BFC/Q	SFC/ α	BFC/Q SFC/ α SAT	BFC/Q	BFC and SFC combined	BFC and SFC combined
Factors varied	speed, tyre tread, resilience, surface	speed, tyre tread, resilience, surface	type of carcass surface	tread wear surface	type of carcass, load, surface	surface
Tyre	1-4	1-4	5-8	9-10	5, 8	1
Surface	1,2 4,5	1,2 4,5	1-4	1,2	4,5	1,2
Speed, km/h (mile/h)	16-48 (10-30)	16-97 (10-60)	16 (10)	48 (30)	48 (30)	48 (30)
Load, kN (lbf)	2.23 (500)	2.23 (500)	2.23 (500)	2.23 (500)	1.78-3.56 (400-800)	2.23 (500)
Inflation pressure, bar (lbf/in ²)	1.1 (16)	1.1 (16)	1.1 (16)	1.1 (16)	1.37 (20)	1.1 (16)
Presented in Tables: Figures:	5,7,9,10 1,2	8 4,5	6 3,6,7		8,9,10; 11,12,13	11

* The abbreviations are BFC (braking force coefficient), SFC (sideway force coefficient), SAT (self aligning torque), α (slip angle), Q (braking slip), 'BFC/Q' means 'braking force as a function of braking slip'.

analysis. For example ancillary programs have been written to provide a wide range of graphs of the results, using an on-line incremental plotter.

4.2 Estimate of error

Errors inherent in the calibration procedure were assessed by evaluating calibration records as if they were test results. In these cases comparison of true and calculated values of braking force coefficient and sideways force coefficient was possible. The results of this comparison for two tyres of different construction are given in Table 4, where error is defined as the difference between the true and the calculated coefficients. It can be seen that the 95 per cent confidence limits for errors are about ± 0.025 in either coefficient. It was not possible to assess the errors under dynamic conditions. However a small sample of repeat runs showed differences lying within ± 0.025 in coefficient. These errors do not of course include any due to variation in the road surface. Some of the results show more scatter than indicated here.

5. RESULTS

5.1 Pure braking force as a function of braking slip

The typical curve of braking force coefficient against braking slip (BFC/Q) – several based on the data of Table 3, Group 1 are shown in Figs. 1 and 2 – has a rapid initial rise and then becomes gradually less steep as it approaches a peak at round about 7 to 25 per cent slip. Beyond the peak there is usually a small fall followed perhaps by a steady or slowly falling region and finally a more rapid fall from 80 or 90 per cent slip to the locked wheel value at 100 per cent slip. Although the curves in Figs. 1 and 2 have been drawn by eye through sometimes rather scattered points they do show genuine differences of shape which arise from the differences of speed, road surface texture, and tyre tread pattern and resilience. The initial rise probably depends on the tyre rather than on the surface since for any given tyre it appears to be common to all surfaces and speeds. Where the peak coefficient is low only a small part of this common curve appears, but the higher the peak the greater the amount of the common curve included. This part is generally understood to be determined by the elastic properties of the tyre carcass. There is a suggestion in Fig. 1 that the initial slope may be steeper for surface 5, the smooth polished mastic asphalt, but such an appearance could be due to very small errors in the measurement of slip or coefficient. The position of the peak coefficient is often very difficult to determine as the curves are often very flat and the experimental points rather scattered in the peak region.

The main characteristics of the curves as affected by speed, surface, tyre tread pattern and resilience are summarised in numerical terms in Table 5. They are also discussed in the later section on the Statistical Analysis of Results (5.5).

5.1.1 Speed and surface Except for the initial sharply rising part speed lowers the curve generally, and lowers the locked wheel value proportionally faster than the peak. The value of Q for the peak BFC seems little affected (its position is often uncertain anyway). The effect of speed in reducing BFC value is much more marked on surfaces where the drainage is poor, that is surfaces which are smooth textured, and for the same reason, with tyres that are smooth. At speeds higher than those investigated here the coefficients fall off still further⁵, only rough textured surfaces or harsh surfaces with patterned tyres giving other than very low coefficients at speeds in excess of

TABLE 4
The error in a single reading of a coefficient derived from static calibrations

Tyre	Coefficient	Mean error	Standard deviation error	95% confidence limits of error	Number of comparisons
Dunlop C41	Braking force	0.005	0.010	-0.015 to +0.025	296
	Sideway force	0.003	0.009	-0.014 to +0.021	296
Michelin X	Braking force	0.008	0.009	-0.009 to +0.025	292
	Sideway force	-0.001	0.009	-0.018 to +0.016	292

TABLE 5
Summary of data of Group 1 of Table 3. Averages of all results at the indicated level of a variable

Quantity	Speed, km/h			Surface character				Tyre character			
				Micro-texture		Macro-texture		Resilience		Pattern	
				Harsh	Polished	Rough	Smooth	Normal	Low	Smooth	Patterned
	16	32	48								
Peak BFC	0.68	0.62	0.59	0.83	0.43	0.58	0.68	0.58	0.69	0.56	0.70
Locked wheel BFC	0.54	0.46	0.38	0.63	0.29	0.43	0.49	0.42	0.50	0.40	0.53
Q for peak BFC	16	16	15	17	15	13	18	15	16	15	17
BFC for Q = 70	1.18	1.25	1.27	1.18	1.30	1.18	1.30	1.22	1.27	1.28	1.20
BFC for Q = 100											
Peak BFC/locked BFC	1.26	1.35	1.55	1.32	1.48	1.35	1.39	1.38	1.38	1.40	1.32
Peak BFC-locked BFC	0.14	0.16	0.19	0.20	0.14	0.15	0.19	0.16	0.19	0.16	0.17

110 – 130 km/h (70 – 80 mile/h).

As already mentioned a harsh surface is capable of giving a high coefficient. To attain it however, a good contact must be made with the surface, and to do this the surface must be rough or the tyre tread pattern must be good or the speed must be low. In the tests of Group 1 the highest coefficients were attained about equally on surface 1 and surface 2, both harsh but surface 1 rough textured in comparison with surface 2.

The road surfaces used in this investigation vary, as has been said, in the two important aspects of (a) macro-texture (rough or smooth), and (b) micro-texture (harsh or polished). Micro-texture has the largest effect on the coefficient in a comparison of one surface with another. Harsh texture gives high coefficients but it can be swamped by the absence of macro scale as speed increases. Roughness or smoothness is less important in itself except as it interacts with speed as already explained. As far as the shape of the BFC/Q curve is concerned there seems from Table 5 little overall effect as far as the ratio of peak to locked wheel value is concerned, and most of the curves look much alike, the biggest difference being in the smooth textured surface 5 where the fall off from the peak with a patterned tyre does seem much more pronounced at the higher speed than for other surfaces.

When one considers that the sliding speed varies from almost zero to the full running speed between the ends of these curves it seems remarkable that there is so little variation in coefficient apart from the initial rise and it is certainly not true that the same coefficient is obtained for the same sliding speed, although in one or two cases where a considerable variation with slip occurs something like this is true. The running speed is clearly of considerable importance as well as the sliding speed. In the case of combined cornering and braking forces it is possible to compare different situations which have the same running and sliding speeds and the agreement between coefficients is then much closer as will be seen later.

5.1.2 Tread pattern, resilience and tyre construction A patterned tyre generally gives a higher coefficient than a smooth one especially at higher speeds; exceptions can sometimes occur at low speeds when the surface is rough and harsh. There is a suggestion in the curves and Table 5 that the peak comes at a slightly larger value of slip with the patterned tyre.

The tyres of lower resilience gave higher coefficients under all circumstances than those of higher resilience, but the shape of the BFC/Q curve was not much affected.

It may be appropriate to mention at this point the effect of crossed ply or radial ply construction on the BFC/Q curve. The tyres of Group 3 were examples of crossed or radial ply tyres rather than matched sets differing only in the form of construction all other things being maintained constant. It will be seen from Fig. 3 that the tyres of radial ply construction did have a more rapid initial rise than the crossed ply tyre. Apart from this the curves differ only slightly and the differences may well be due to causes other than that of carcass construction. As far as peak coefficients are concerned a comparison of the two forms of construction with other factors evenly matched has shown that the radial ply gives larger values, sometimes up to 0.1 greater. The locked wheel coefficient for the radial ply tended however to be higher on smooth textured and lower on rough textured surfaces, the differences being smaller, up to 0.05 in coefficient⁶. The more rapid initial rise here

found for the BFC/Q curve is well known to occur with SFC/ α curves. Similarly an increased slope has been found for the BFC/Q curve of a worn tyre; such a tyre has lost a certain degree of flexibility in losing the thickness of the tread.

It may be noted, as an example of the complicated interactions of the various factors involved, that the most extreme radial ply tyre No. 8 gave higher peak BFC's on the harsh surfaces 1 and 2 than the crossed ply tyre No. 5, but that on the polished surfaces 3 and 4 gave lower values. The same was true less obviously of the locked wheel BFC.

Reference will be made to the Group 1 data again in the section devoted to a statistical analysis of the data (5.5.1).

5.2 Pure cornering force as a function of slip angle

Typical SFC/ α curves from the data of Group 2 are shown in Figs. 4a and 4b. The initial rise is closely linear, then the curve rises steadily but with decreasing slope to a maximum whose position is quite clearly dependent on the magnitude reached and then falls off relatively slowly. As for the curves of BFC/Q, the initial part of the curve is largely independent of the surface, tread pattern and resilience, except perhaps at the higher speeds where even the peak values are low and the curves relatively rather uncertain. In Figs. 4a and 4b owing to doubts originally as to the meaning of observations the curves have all been shifted so as to pass through the origin, whereas several of them as recorded did not do so, a cornering force having been obtained at zero angle of slip. The displacements necessary ranged up to 9 mrad (about 0.5°), corresponding to a coefficient of about 0.1 or a cornering force of 220 N (50 lbf). The existence of such zero angle cornering forces has been reported elsewhere.

5.2.1 Speed and surface Like the BFC's of Group 1 the SFC's of Group 2 have been examined over the same group of surfaces and tyres, and so vary with speed, surface, tyre tread pattern and resilience. The general relationships with speed and surface are as for the BFC; and for peak coefficients some are illustrated in Fig. 5, the speed range being greater than for the BFC. It should again be noted that owing to the overlap of the initial parts of the SFC/ α curves the SFC at small angles of slip may well be independent of speed up to a certain limit depending on the slip angle and the surface etc.

5.2.2 Tread pattern, resilience and tyre construction In general, the patterned tyre gives greater values of SFC than the smooth; similarly, tyres of low resilience tread rubber give greater values than those of high resilience tread rubber. That the dependence on tyre tread pattern and resilience can be complex is shown by some of the curves of peak value in Fig. 5 where for example pattern improves the peak value on surface 1 with tyres 3 and 4 but not with tyres 1 and 2. The same is true on surface 2 below about 45 km/h but not above it.

As is so well known, the radial ply tyre tends to have a steeper initial slope than an equivalent crossed ply. Figure 6 and Table 6 show the results of the comparison of radial and crossed ply tyres. It should be remembered that these tyres differ in respect of tread pattern, resilience etc., as well as type of construction so that not all the detailed differences are due to the latter. The Laboratory has obtained some evidence that as far as peak and locked wheel BFC's are concerned matched or

TABLE 6

Comparison of Results for Crossed Ply and Radial Ply Tyres (Data of Group 3 Table 3)

Tyre	Surface	SFC at 25 mrad * (1.4°)	Peak SFC	Peak BFC	Locked wheel BFC	Q for peak BFC
5 crossed ply	1	0.16	0.84	0.73	0.53	16
	2	0.15	0.81	0.86	0.69	16
	3	0.17	0.47	0.48	0.38	14
	4	0.19	0.38	0.40	0.28	14
	mean	0.17	0.63	0.62	0.47	15
6 textile banded radial	1	0.20	0.84	0.71	0.53	16
	2	0.21	0.83	0.88	0.68	20
	3	0.20	0.49	0.50	0.39	14
	4	0.21	0.39	0.41	0.30	14
	mean	0.21	0.64	0.63	0.48	16
7 textile banded radial	1	0.19	0.80	0.67	0.57	12
	2	0.20	0.89	0.85	0.71	10
	3	0.20	0.50	0.47	0.38	8
	4	0.23	0.41	0.39	0.27	10
	mean	0.21	0.65	0.58	0.48	10
8 metal banded radial	1	0.25	0.85	0.82	0.53	16
	2	0.28	0.89	0.90	0.76	17
	3	0.21	0.53	0.44	0.34	15
	4	0.18	0.42	0.35	0.27	10
	mean	0.23	0.67	0.63	0.48	15

* This quantity is very difficult to estimate from plotted data

nearly matched tyres do show some differences in the peak and locked wheel values, which confirm in general the evidence of higher peak values for the radial ply tyres⁶. This has been mentioned earlier in connection with BFC's.

Previous researchers have found that the initial slope of the SFC/ α curve may increase by 20 per cent when the tread depth is worn down to 2.5mm⁷ or less⁸. As the difference in initial slope between tyres 5 and 6 on surface 1 is of the order of 25 per cent, see Table 6, a worn crossed ply tyre might have an initial slope (i.e. cornering stiffness) comparable with that of a new textile ply radial tyre in certain circumstances. Such changes due to wear would seem to be important from the point of view of car handling.

5.3 Self aligning torque in pure cornering

Some data on the relations between the self aligning torque (SAT) and the slip angle or the sideways force coefficient are shown in Fig. 7; they are derived from the Group 3 data on the comparison of tyre construction types. The self aligning torque rises to a peak at a point where the SFC is still rising and where it has reached something like half or rather more than half of its peak value, then diminishes to zero at a point where the SFC has reached or is approaching its maximum and then becomes negative. According to the selection of curves in Fig. 7 the slopes of the SAT/ α curves are in the same order of magnitude as the cornering stiffnesses. On the other surfaces studied where the values were smaller it is not possible to say whether the same holds true, but the general shapes of the curves were very similar to those shown. When SAT is plotted against SFC the initial slope seems less dependent on cornering stiffness as might be expected.

The fact that the SAT change from positive to negative is fairly closely associated with the maximum of SFC suggests that this changeover might possibly be used in a control system for the cornering force.

5.4 Combined action of braking and cornering forces

When braking and cornering forces act simultaneously, and are therefore both competing for the available friction, one would expect from simple dry friction theory that the one would not affect the other until limiting friction was attained and that beyond that point the forces would be distributed in accordance with the direction of the slip or slip velocity. In the more complicated condition of tyre friction only an approximation to this behaviour can be expected.

5.4.1 Effect of braking on the sideways force coefficient/slip angle curve Figures 8 and 9 show the effect of braking force and braking slip on the relation between sideways force coefficient and slip angle. For a given braking force coefficient the initial rate of rise of the curve appears to be largely unaffected, then the curve breaks away and reaches a lower maximum than before. As the amount of braking force is increased this maximum is reduced in value but may become more pronounced a feature than in the original curve. At values of BFC greater than the locked wheel value the curves become closed loops, one of which is shown in Fig. 8, thus the frictional demand of a large braking force cannot be satisfied at slip angles greater than a certain amount.

The lower branch of the loop has its counterparts near the bottom of the figure where for example a second curve corresponding to a BFC of 0.3 occurs. These lower curves are presumably

TABLE 7

Regression Equations of Braking Force Coefficient as a Function of Braking Slip, Speed, Resilience and Pattern (Significant at 5% level)

Surface	a_0 constant	$a_1 \times 10^2$ V	$a_2 \times 10^4$ V^2	$a_3 \times 10^2$ Q	$a_4 \times 10^4$ Q^2	$a_5 \times 10^6$ Q^3	$a_6 \times 10^2$ Re	a_7 P	Degrees of freedom	σ
1	0.804	*	-0.287	1.572	-3.470	1.967	-0.405	*	196	0.086
2	0.860	*	-0.518	1.811	-3.583	1.879	-0.509	*	198	0.085
4	0.465	-0.118	*	*	-0.190	0.120	-0.306	0.059	204	0.031
5	0.859	-1.813	1.510	0.932	-2.013	1.078	-0.498	0.300	196	0.074

* denotes coefficient not significant

TABLE 8

Regression Equations of Sideway Force Coefficient as a Function of Cornering Slip Angle, Speed, Resilience and Pattern (Significant at 5% level)

Surface	a_0 constant	$a_1 \times 10^2$ V	$a_2 \times 10^4$ V^2	$a_3 \times 10^2$ α	$a_4 \times 10^4$ α^2	$a_5 \times 10^6$ α^3	$a_6 \times 10^2$ Re	a_7 P	Degrees of freedom	σ
1	0.131	-0.069	*	0.952	-0.329	0.036	-0.396	*	451	0.057
2	0.167	*	0.261	0.912	-0.336	0.039	-0.459	0.116	451	0.106
4	0.275	-0.124	0.061	0.450	-0.229	0.034	-0.459	*	415	0.035
5	0.578	-0.958	0.442	0.432	-0.171	0.021	-0.610	0.171	413	0.083

* denotes coefficient not significant

lower branches of loops which because they extend outside the slip angle range covered by the figure do not therefore appear as loops. These lower curves correspond to values of BFC beyond the peak which would appear in a curve of BFC against braking slip (Q) for a fixed slip angle, see for instance Fig. 11, and therefore represent conditions in which a brake would be unstable and tend to lock. In Fig. 8 the line dividing the unstable and stable regions, which joins the points which are points of maximum BFC for a given slip angle, is shown. After an initial fairly rapid rise it is almost horizontal at about an SFC of 0.1 on this polished low friction surface.

Figures 8 and 9 also contain curves for constant braking slip Q as well as for constant BFC. These, the dashed curves, are quite different from those of constant BFC. The initial slope is affected markedly but the curves rise to maxima which though outside the range examined would appear to be much less affected when Q is small than the earlier (left-hand) parts of the curves.

The negative value of SFC obtained with the locked wheel at small slip angle is comparable with the error of measurement and may have no other significance.

5.4.2 Effect of braking slip on SFC at a given slip angle Figure 10 shows this effect. The cornering force falls off very rapidly with braking slip. On the whole the curves for various slip angles look fairly parallel at first so that the reduction due to a given amount of braking slip is roughly the same up to braking slips of about 30 per cent whatever the initial figure. Some of the curves suggest that the maximum SFC does not occur at zero braking slip but the discrepancies are all very small and may possibly be due to error, though this phenomenon has been noted elsewhere¹.

5.4.3 Effect of slip angle on the braking force coefficient/braking slip curve Figure 11 shows the effect of a fixed slip angle on the BFC/ Q curve. The initial very rapid rise, characteristic of the pure BFC curve, disappears and the maximum moves towards the right and might be said to disappear altogether except that the fall near to 100 per cent braking slip still occurs.

Curves of BFC/ Q for fixed values of the cornering force would be very similar to those of Fig. 11 as far as the initial rise is concerned, as may be seen from Fig. 12 in which it is clear that curves of constant slip angle are, at least initially, curves of approximately constant sideways force also.

5.4.4 Other aspects of combined forces Another form in which the mutual action of combined forces has been expressed is shown in the Fig. 12 already mentioned, where BFC and the corresponding SFC are plotted for fixed values of slip angle. It has been suggested that such curves should be quadrants of ellipses of fixed horizontal axis lying within a bounding circular quadrant⁹. It does not appear from Fig. 12 that these expectations are fulfilled. Although peak values are sometimes rather uncertain, it seems clear that for tyre 5, a crossed ply tyre, the pure braking force peak is smaller than the pure sideways force peak. For tyre 8, a radial ply tyre, the suggestion is much more plausible, although the divergences in the case of the lowest and highest loads would seem to be real. The curves in Fig. 12 do suggest however that beginning from BFC = 0 they were aiming at the bottom right hand corner of the quadrant but that they deviate from it because of a progressive reduction in the resultant force. If the right hand end points of the curves corresponded to the locked wheel i.e. to a braking slip of 100 per cent, they should lie on lines from the origin at a slope of $\tan \alpha$. The curves for 2670 N (600 lbf) loading, which terminate at $Q = 100$, confirm this hypothesis for the larger values of α . Frequently the SFC is very small or even negative for $Q = 100$. The

magnitudes of the SFC in these cases are very small, comparable with the expected range of error and whether they are real or simply errors of measurement is uncertain.

On the other hand as already mentioned, it is known that there may be a sideforce when a tyre runs at zero slip angle, so that the curves for zero slip angle do not necessarily lie along the horizontal axis.

5.4.5 Resultant force and slip In Fig. 12 the resultant force coefficient (RFC) at the tyre contact is represented by the radius vector from the origin to the point on the curve. This quantity tends to remain fairly constant in value provided the slip angle is sufficient for the SFC to be close to its maximum in the absence of braking force. The sharp curvature at the ends of some of the graphs in Fig. 12 corresponds to the rapid fall off as the locked wheel condition is approached. The results are compatible with this fall off occurring in the cornering force as well as in the braking force but the magnitudes are too small for it to be possible to assert that this does occur.

Figure 12 gives simultaneous values of BFC, SFC and RFC but does not show the amount of slip involved. It is of interest to plot, as in Fig. 13, all three coefficients against the amount of slip involved: the definition of resultant slip used in the combined case being given in Appendix 1. It is to be noted that curves of BFC/Q and SFC/ α plot very similarly in relation to the slip. An earlier investigation using a smooth tyre carried out on six surfaces showed that close agreement was obtained between braking force and sideways force tests when the results were compared at the same sliding speed¹⁰.

It should be remembered that the actual slip under the tyre is complex and that the slip as calculated may not bear the same relationship to the actual slip for pure SFC as it does for pure BFC and consequently for intermediate combined cases also. One would expect the best agreement when both rates of slip were high, i.e. well beyond the value giving peak forces. If the correct slip was known an even better fit might possibly have been obtained. However even apart from this Fig. 13 shows a specially favourable case, since Fig. 12 shows that in this case the peak pure BFC and SFC were very nearly equal. Had some of the other cases in Fig. 12 been chosen for this treatment a discrepancy in the peak values would have appeared, and agreement would certainly have been worse. The agreement, such as it is, certainly seems to depend on tyre conditions such as construction, tread and loading.

5.5 Statistical analysis of results

The large number of results obtained in investigations such as Groups 1 and 2 in which several factors are varied makes it difficult to present them economically or to discover their full implications. Comparison of large numbers of curves usually reveals few regularities free from anomalies. Statistical methods seem best adapted to cope with the situation. A multiple regression technique was therefore used in an attempt to condense the results into more manageable form. The work to be described was of a somewhat exploratory nature and the main purpose was to derive the dependence of the coefficients on speed, slip or slip angle, tyre tread pattern and resilience. The differences between surfaces and the many interactions between the factors were not investigated. A fuller treatment requires that the characteristics of surfaces be included and those of tyres more fully represented by suitable parameters. Investigations into these are under way.

5.5.1 Braking force coefficient The Group 1 data on braking force as a function of braking slip, vehicle speed, tyre tread resilience and pattern were assumed to be represented by a regression equation of the form:

$$\mu = a_0 + a_1 V + a_2 V^2 + a_3 Q + a_4 Q^2 + a_5 Q^3 + a_6 Re + a_7 P$$

where a_0, a_1, a_2 etc. are constants

V = vehicle speed (km/h)

Q = percentage braking slip

Re = tyre tread resilience (percentage rebound on Lupke resiliometer)

P = tyre tread pattern, given the value 0 for a smooth tyre and 1 for a patterned tyre.

Re was in the range 31 to 55. Percentage braking slip varied from 0 (free rolling wheel) to 100 (locked wheel). The speed ranged from 16 to 48 km/h.

Calculations were done by the Pegasus computer and the results at the 5% significance level are given in Table 7, which also gives details of the degrees of freedom and the root mean square residual (σ). It will be seen that σ is so large on all except surface 4 as to discourage the use of this regression equation as a direct substitute for the experimental results. The number of terms in Q is inadequate to deal with the complexity of the dependence on slip. The equations do however give an indication of the relative effects of vehicle speed, tyre tread resilience and tread pattern on the coefficient. It may be noted that the effect of tyre tread resilience is quite large and significant on all four surfaces, whereas the effect of tread pattern is not significant on two surfaces but is very large on surface 5 as is the effect of vehicle speed. The reason for the low value of σ on surface 4 is that this surface in the wet has low fairly uniform coefficients of friction under most conditions whereas on the other surfaces the coefficients are much less uniform.

5.5.2 Sideway force coefficient The dependence of sideway force coefficient on the other variables for the Group 2 data, was assumed to be of the form:

$$\mu = a_0 + a_1 V + a_2 V^2 + a_3 \alpha + a_4 \alpha^2 + a_5 \alpha^3 + a_6 Re + a_7 P$$

where α is cornering slip angle measured in mrad, and the other variables are as before. Vehicle speed now ranges from 16 to 97 km/h, and cornering slip angle from 0 to 350 mrad. Other parameter ranges were as for the BFC/Q work.

Table 8 gives the results of regression analysis. Again σ is mostly rather large, dependence on slip angle not being adequately dealt with, and so discourages the use of the equation as a direct substitute for experimental results. As for BFC the effect of resilience is significant and quite large on all surfaces. Tread pattern makes a notable contribution on surfaces 2 and 5, both of which are smooth textured. Surface 5 again has a very large drop in coefficient with increase of vehicle speed; it is both smooth and polished.

Comparison of the values predicted from regression equations with experimental results show that to use cubic terms in wheel slip or slip angle, does not allow a very good fit. A much better

fit (σ about 0.035) to the slip dependence can be obtained by using quintic polynomials, but owing to the spread in the experimental points such curves can differ widely from curves which would be drawn by eye.

6. CONVERSION OF DATA INTO PRACTICAL QUANTITIES: A STUDY OF BRAKING

The various characteristics of the data under investigation are of importance in practice through the effects they have on the braking, handling etc, of vehicles. A means of simplifying the data and showing up their practical significance or insignificance may therefore be to convert them theoretically into such practical terms. The present section deals with the conversion of braking force characteristics into braking distances, wheel locking times etc. for a vehicle under different types of braking. The work may also be considered in its own right as a theoretical study of wheel behaviour for BFC/Q relations of the type obtained in these investigations.

6.1 Theoretical Model

A typical vehicle is represented for this purpose by a single wheel and a mass (including that of the wheel) of 227 kg (500 lb). It is assumed to be braked in a straight line. Appendix 2 contains the equations of motion, the assumptions made, and outlines the method of solution on the Pegasus computer.

For a tyre of given tread resilience and tread pattern the braking force coefficient was assumed to be a function of vehicle speed and percentage braking slip. The data of Group 1 for four wet surfaces and for tyres with differing tread patterns and tread resiliences were used. The speed ranged from 16 to 48 km/h (10 to 30 mile/h), and the percentage braking slip from 0 to 100. The motion was therefore assumed to start at time $t = 0$ with the wheel free rolling and a vehicle speed of 48 km/h (30 mile/h). Brake torque (Φ) was assumed applied according to some function of time and the resulting motion computed until the linear speed was zero. Brake torque inputs considered were as follows:

- 1) $\Phi = A$ (constant); a constant torque applied instantaneously at $t = 0$
- 2) $\Phi = \frac{At}{t_R}$ for $t \leq t_R$, otherwise $\Phi = A$; a brake torque applied with finite rise time t_R ,
- 3) $\Phi = A | \sin \pi f t |$, a brake torque pulsed on and off at f cycles per second with amplitude A and sinusoidal shape. In this case when vehicle speed (V) drops to about 1.22 m/s the brake torque is set so that the wheel locks until $V = 0$.

The program output consisted of values of time, distance travelled, vehicle speed, angular velocity of wheel, percentage braking slip, brake torque and braking force coefficient. The numerical method was arranged to give the instant of wheel locking (when this occurred) quite accurately by time interval changing. When the wheel locked the time interval in the numerical method was made larger so cutting computer running time without significant loss of accuracy. Small time intervals are needed to get accurate results when the wheel is rotating.

If the braking force coefficient could be maintained at the peak value of the BFC/Q curve (the ideal anti-locking case) it would be a function of vehicle speed only, so that the theoretical minimum

straight line braking distance can easily be computed as shown in Appendix 2. This minimum braking distance may be usefully compared with the braking distance implied by a proposed antilocking device.

6.2 Results

Table 9 shows the results obtained in terms of braking distance and time taken for the wheel to lock for braking with a variable rise time t_R ; it also gives for comparison the theoretical minimum braking distance. It can be seen that there is a wide variation in braking distance depending upon the surface and tyre properties. It is of interest to note that changes in t_R do not affect braking distances very much, but they do change quite considerably the time taken for the wheel to lock.

Table 10 shows the time taken to reach the peak BFC compared to the time taken for the wheel to lock for different values of t_R . It will be seen that the peak is passed very quickly in some cases.

For pulsed braking very small time intervals are need for much of the motion, thus requiring a lot of computer running time. However results were obtained for the smooth tyre of resilience 55% on surface 2 (Fine Cold Asphalt), with BFC/Q data as in Fig. 14. The value of A was 949 Nm and the brake pulsed at 3, 5 and 8 cycles per second; the braking distances were 16.9 m, 16.8 m and 17.0 m respectively. Comparison of these values with 19.8 m obtained with constant braking shows the potential benefit of pulsing for this particular combination of surface and tyre. Also it may be noted that the theoretical minimum braking distance is 12.2 m, so that pulsing in this simple way is by no means an ideal form of antilocking.

7. SUMMARY AND CONCLUSIONS

For wet road surfaces the dependence of tyre forces on tyre slip has been examined for braking and cornering forces acting alone and in combination and for self aligning torque. Tyre force data depend on so many factors that it is difficult to present them economically or to analyse them satisfactorily. Statistical methods can contribute substantially towards elucidating the effects of the various factors and their interactions. It is also valuable to convert the data by theoretical means into more practical quantities such as braking distances, or their effects on antilocking devices. A summary of the main features of the results as well as some conclusions which seem of interest in relation to vehicle stability, antilocking braking or tyre design are given below.

(1) Curves of pure BFC (braking force coefficient) against braking slip, and of pure SFC (sideway force coefficient) against slip angle, are rather similar; a rapid rise to a peak followed by a much slower fall off. The initial part of the rise is nearly linear and depends largely on the elastic properties of the tyre carcass and tread; its steepness is thus affected to an important extent by whether the construction is crossed ply or radial. The general level of the curve beyond the peak is largely determined by speed, road surface texture and the pattern and resilience of the tyre tread. The possible variation is large, and the peak value may easily range from almost 0.0 to 1.0. For BFC, the available curve extends to 100 per cent slip (locked wheel) and the value of BFC for that point is not only smaller than the peak value but is definitely less than the value at a slip of about 80 or 85 per cent.

(2) When braking and cornering forces act simultaneously the curves referred to in (1) are

TABLE 9

Braking Distances and Wheel Locking Times for Varying t_R and $A \approx 949 \text{ Nm}$

Surface	Tyre		Braking distance, m			Locking time, ms			Theoretical minimum braking distance, m (X)
	Tread	Resilience *	$t_R=0$	$t_R=0.1s$	$t_R=0.3s$	$t_R=0$	$t_R=0.1s$	$t_R=0.3s$	
1	smooth	55	18.4	18.5	18.5	100	174	327	11.3
	smooth	38	14.8	15.1	15.4	112	185	341	10.7
	patterned	51	15.3	15.6	16.0	102	172	322	12.2
	patterned	31	13.8	14.1	14.5	127	201	363	9.8
2	smooth	55	19.8	19.8	19.7	97	168	318	12.2
	smooth	38	14.5	14.8	15.1	111	182	339	10.5
	patterned	51	14.0	14.3	14.8	108	180	334	11.3
	patterned	31	10.8	11.2	11.9	163	239	407	8.8
4	smooth	55	43.9	44.0	43.9	64	121	228	32.8
	smooth	38	39.7	39.8	39.9	65	123	231	28.3
	patterned	51	38.3	38.4	38.4	66	124	235	27.5
	patterned	31	30.7	30.8	30.8	70	132	253	21.0
5	smooth	55	49.4	49.4	49.2	60	114	210	35.6
	smooth	38	48.6	48.6	48.1	61	116	215	29.3
	patterned	51	25.5	25.3	24.8	76	142	276	14.3
	patterned	31	22.2	21.9	21.2	89	161	311	10.9

* Percentage rebound measured on a modified R.S. Lupke pendulum at 20°C

TABLE 10

Time Taken to Reach Peak Braking Force Coefficient and Time Taken to Lock
the Wheel for Varying t_R and $A = 949 \text{ Nm}$

Surface	Tyre		$t_R = 0$		$t_R = 0.1s$		$t_R = 0.3s$	
	Tread	Resilience *	Time to peak BFC, ms	Time to lock ms	Time to peak BFC, ms	Time to lock ms	Time to peak BFC, ms	Time to lock ms
1	smooth	55	10	100	85	174	215	327
	smooth	38	20	112	95	185	240	341
	patterned	51	15	102	85	172	210	322
	patterned	31	20	127	95	201	245	363
2	smooth	55	10	97	75	168	200	318
	smooth	38	25	111	95	182	240	339
	patterned	51	20	108	90	180	225	334
	patterned	31	40	163	115	239	285	407
4	smooth	55	5	64	40	121	95	228
	smooth	38	5	65	50	123	110	231
	patterned	51	10	66	55	124	120	235
	patterned	31	10	70	65	132	140	253
5	smooth	55	10	60	45	114	85	210
	smooth	38	10	61	50	116	100	215
	patterned	51	15	76	70	142	175	276
	patterned	31	15	89	85	161	215	311

* Percentage rebound measured on a modified B.S. Lupke pendulum at 20°C

modified. If a definite braking slip is imposed, the curve of SFC/slip angle has in general a smaller initial slope, a lower peak and attains the peak at a larger angle of slip; for example, the change of slip means that cornering stiffness is reduced to about half by a 10 per cent braking slip in one case quoted. The imposition of small or medium amounts of braking force (which generate little slip) does not much affect the cornering stiffness. In general however, they reduce the maximum SFC and the slip angle at which it occurs. These changes are important in vehicle handling.

(3) The curve of BFC/braking slip is similarly affected by fixed amounts of slip angle and sideways force: with a given slip angle the initial slope is decreased and the peak is attained at a much larger amount of braking slip. These effects are probably more directly relevant to the behaviour of antilocking devices than to vehicle stability.

(4) Only a very small amount of data is available on self aligning torque and only in the absence of braking forces. The torque at slip angles beyond the peak of SFC was found to be negative and larger for the radial ply than for the crossed ply tyres tested.

(5) There is some evidence that when braking and sideways forces are both acting and the resultant coefficient is plotted against the resultant slip for a given tyre, surface and speed, the points tend to lie fairly close to a single line except perhaps for the initial rise, i.e. to a large extent the resultant coefficient is a function of resultant slip, and independent of the type of slip (braking or cornering) or the relative amounts. Resultant slip, is a somewhat arbitrarily defined quantity; if true slip were known an even closer agreement might possibly have been found.

(6) Non-locking braking systems can greatly extend the conditions under which a cornering vehicle will remain stable and controllable during severe braking if they can prevent the available side forces from dropping to low values. Systems which seek to maintain the braking slip between say, 16 and 20 per cent, are likely to be more effective than those that seek for peak braking, for the latter may ensure that the available side force is very small.

(7) Typical braking distances to standstill as calculated from the BFC/slip data appeared to be almost unaffected by the time required to apply full braking torque (range 0–0.3 s), although the times to reach peak braking or lock the wheel were markedly affected. These distances were from 25 to 100 per cent greater than if the braking force could have been kept always at the peak value.

(8) Statistical analysis of some of the data indicated that a low tread resilience increased the coefficients on the polished surfaces as well as on the harsh.

8. APPENDIX I

EVALUATION OF RESULTS AND DEFINITION OF SLIP

8.1 Notation

L	vertical load
B	braking force
S	sideway or cornering force
R	resultant horizontal force
V	vehicle speed
v	theoretical sliding speed of contact patch
q	component of theoretical sliding speed in plane of wheel
Q	braking slip
α	cornering slip angle
ω_t	angular speed of test wheel
ω_v	angular speed of vehicle wheel
r_t	rolling radius of test wheel
r_v	rolling radius of vehicle wheel
k	r_t/r_v
d	effective length of axle shaft
h	distance between upper and lower suspension ball joint centres
u	$r_t\omega_t$ = linear speed of test wheel

8.2 Correction for effect of braking on measured forces

As already mentioned in the main text the application of a braking force at the contact patch when the test wheel is inclined results in additional forces at the measuring system. Because of the characteristics of the wheel and axle the only forces which can be called into action to balance the effect of the braking force couple $r_t B$ on this system, are the vertical forces F_1 and the horizontal forces F , as shown in Fig. A1. Friction in the system is ignored as is the rolling resistance couple. The couples acting on the system are shown in Fig. A2. Equating to zero the resultant components along OX and OY gives

$$\begin{aligned} -r_t B + r_t B \cos \alpha + d F_1 \sin \alpha &= 0 \\ h F + d F_1 \cos \alpha - r_t B \sin \alpha &= 0 \end{aligned} \quad (1)$$

or

$$\begin{aligned} F &= r_t B \tan(\alpha/2)/h \\ F_1 &= r_t B \tan(\alpha/2)/d \end{aligned} \quad (2)$$

Since the forces are measured at the upper ball joint the following corrections are therefore required and were incorporated in the evaluation program:

$$\begin{aligned} \text{True } S &= \text{Observed } S + r_t B \tan(\alpha/2)/h \\ \text{True } L &= \text{Observed } L - r_t B \tan(\alpha/2)/d \end{aligned} \quad (3)$$

8.3 Definition and evaluation of braking slip Q

Figure A3 shows the linear and angular speeds of the test wheel and vehicle wheel, u would be the linear speed of the test wheel if it were rolling with angular velocity ω_t and q is the assumed theoretical sliding speed of the contact patch in the plane of the wheel. Then we have:

$$\begin{aligned} V &= r_v \omega_v \\ u &= r_t \omega_t \end{aligned} \quad (4)$$

and q is defined by

$$V \cos \alpha = q + u$$

The braking slip Q is then defined by

$$Q = 100q/V \cos \alpha = 100(1 - (u/V \cos \alpha))$$

and finally

$$Q = 100 (1 - (k \omega_t / \omega_v \cos \alpha)) \quad (5)$$

where $k = r_t / r_v$. Q is therefore 100 times the ratio of the component of the theoretical sliding speed in the plane of the wheel to the component if the wheel were locked. This definition makes the braking slip 100 per cent for a locked wheel, even though the sliding velocity in the plane of the wheel is not 100 per cent of the linear velocity V .

Quantities inversely proportional to ω_t and ω_v were recorded during test and their values used in (4) and (5) to evaluate Q and V . The value of k was determined at the beginning of a series of tests with the particular tyre.

8.4 Resultant force and resultant slip

R , the resultant horizontal force is simply the resultant of the braking and sideways forces B and S and is given by $R^2 = B^2 + S^2$.

A resultant sliding speed v was defined as follows. The sliding speed in the plane of the wheel was assumed to be q or $QV \cos \alpha / 100$ and the component perpendicular to the plane of the wheel to be $V \sin \alpha$. The resultant sliding speed v is then given by

$$v^2 = V^2 \sin^2 \alpha + Q^2 V^2 \cos^2 \alpha / 10000 \quad (6)$$

A resultant slip would be defined as $100v/V$ and its component in the plane of the wheel for the locked case would not be 100, whereas the braking slip Q would be 100 as defined.

The directions of the resultant force and resultant sliding speed were also computed as a check on how closely they coincided, especially in the case of the locked wheel when they should coincide exactly.

9. APPENDIX 2

THEORY OF BRAKED WHEEL REPRESENTING A BRAKED VEHICLE

9.1 Notation

μ	braking force coefficient, a function of V and Q
B	$= \mu Mg =$ braking force
M	mass including that of wheel
I	moment of inertia of wheel about its axis
V	linear speed of wheel
V_0	initial linear speed of wheel
s	distance travelled
X	theoretical minimum braking distance
ω	angular speed of wheel
r	rolling radius of wheel, assumed constant
Q	$= 100 (1 - (r\omega/V)) =$ braking slip
g	acceleration due to gravity
t	time
Φ	applied brake torque

9.2 Theory

A consistent set of units such as the 'SI' Metric is assumed. The equations of motion for the wheel and mass as shown in Fig. A4 are:

$$\begin{aligned}
 \frac{ds}{dt} &= V \\
 \frac{dV}{dt} &= -\mu g \\
 \frac{d\omega}{dt} &= (\mu Mg r - \Phi)/I
 \end{aligned}
 \tag{1}$$

In (1) μ is defined as follows. For Q between 5 and 100 per cent μ is a function of slip and speed of the form

$$\mu = f_1(Q) + f_2(V) \tag{2}$$

The function $f_1(Q)$ is defined by a table of values derived from an empirical curve representing μ for the initial speed V_0 , like that shown in Fig. 14. The function $f_2(V)$ is a quadratic function of V

$$f_2(V) = b_0 + b_1 V + b_2 V^2 \tag{3}$$

where the constants b_0 , b_1 and b_2 were derived from the statistical analysis of the data and

$$f_2(V_0) = 0. \tag{4}$$

For Q between 0 and 5 per cent, it is assumed that at any V the curve of μ against Q is a straight line joining the origin to the 5 per cent value, as shown in Fig. 14.

The equations (1) subject to (2), (3) and (4) were solved on the Pegasus computer, by using a fourth order Runge-Kutta integration routine. Interval changing was written into the program in order to determine accurately the instant when the wheel locked or (when the brake was pulsed) unlocked. It was arranged to have large time steps when the wheel was locked in order to economise on computer running time. Interval changing was again used to bring V to zero without overshooting. By the definition of percentage slip, when V is small a small change in ω will produce a large change in Q . For this reason when the brake was pulsed the pulsing was stopped when V reached about 1.22 m/sec and the brake torque was then kept constant and large enough to lock the wheel. There was very little loss of accuracy in doing this.

In order to compute the theoretical minimum braking distance it is assumed that μ is a function $\mu(V)$ of V only, thus ignoring the time taken for peak coefficient to be attained. Then the equation for linear motion is:

$$\frac{dV}{dt} = -\mu(V)g$$

or
$$V \frac{dV}{ds} = -\mu(V)g$$

Hence
$$\left[gds \right]_0^x = \int_{V_0}^0 \frac{-VdV}{\mu(V)}$$

i.e.
$$X = \frac{1}{g} \int_0^{V_0} \frac{VdV}{\mu(V)} \quad (5)$$

It should be noted that $\mu(V_0)$ is the peak coefficient of the BFC/ Q curve for $V = V_0$. Equation 5 can be integrated by standard methods and so X is obtained.

10. APPENDIX 3

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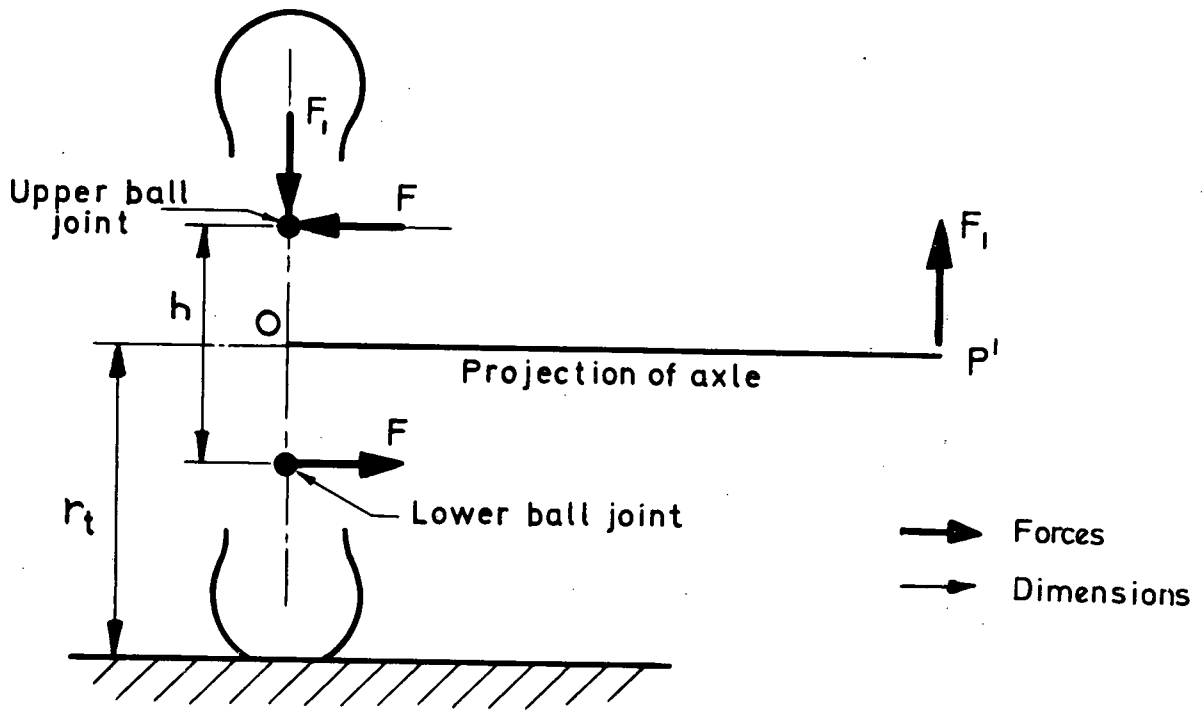


Fig.A1. PROJECTION OF SYSTEM ON ZX PLANE (See Fig. A2 for axis systems) SHOWING FORCES REQUIRED TO BALANCE THE COUPLE DUE TO THE BRAKING FORCE

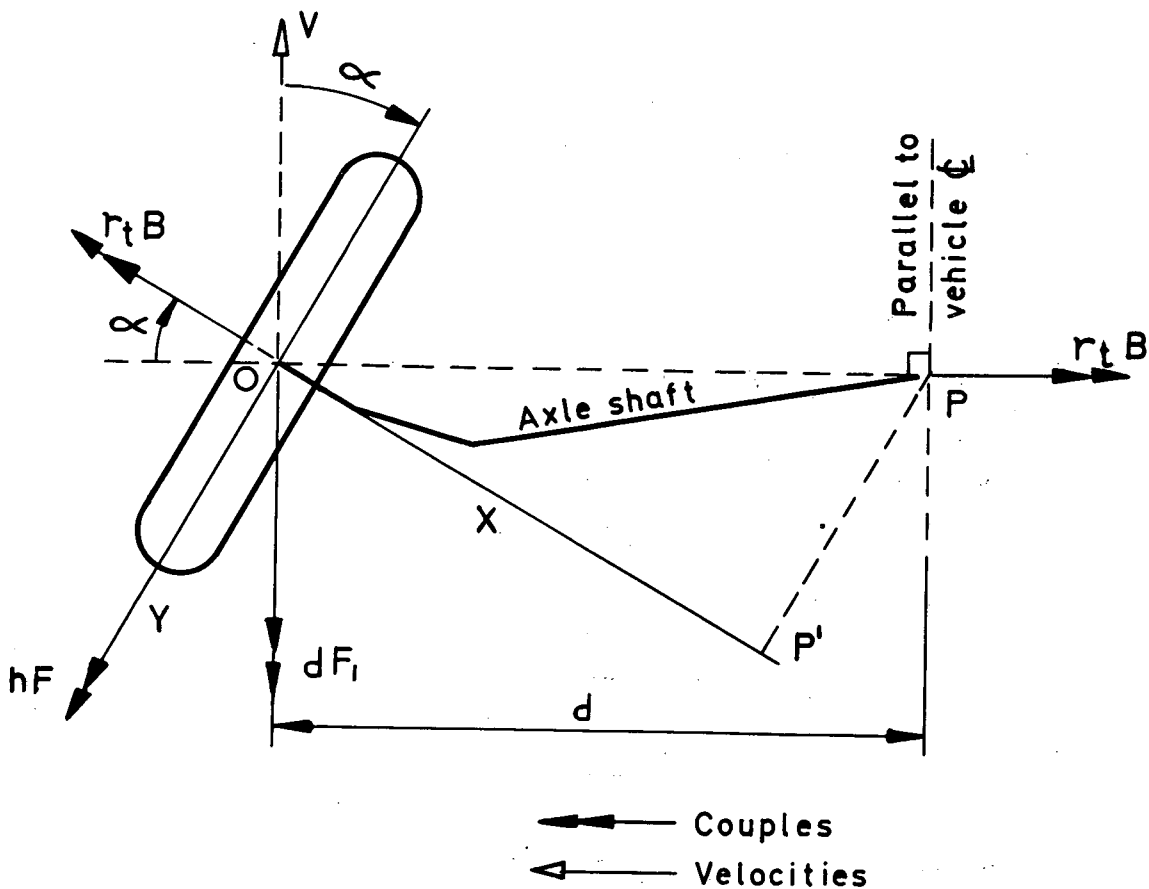


Fig.A2. PLAN VIEW OF AXLE SHAFT AND TEST WHEEL SHOWING COUPLES ACTING ON AXLE AND WHEEL SYSTEM

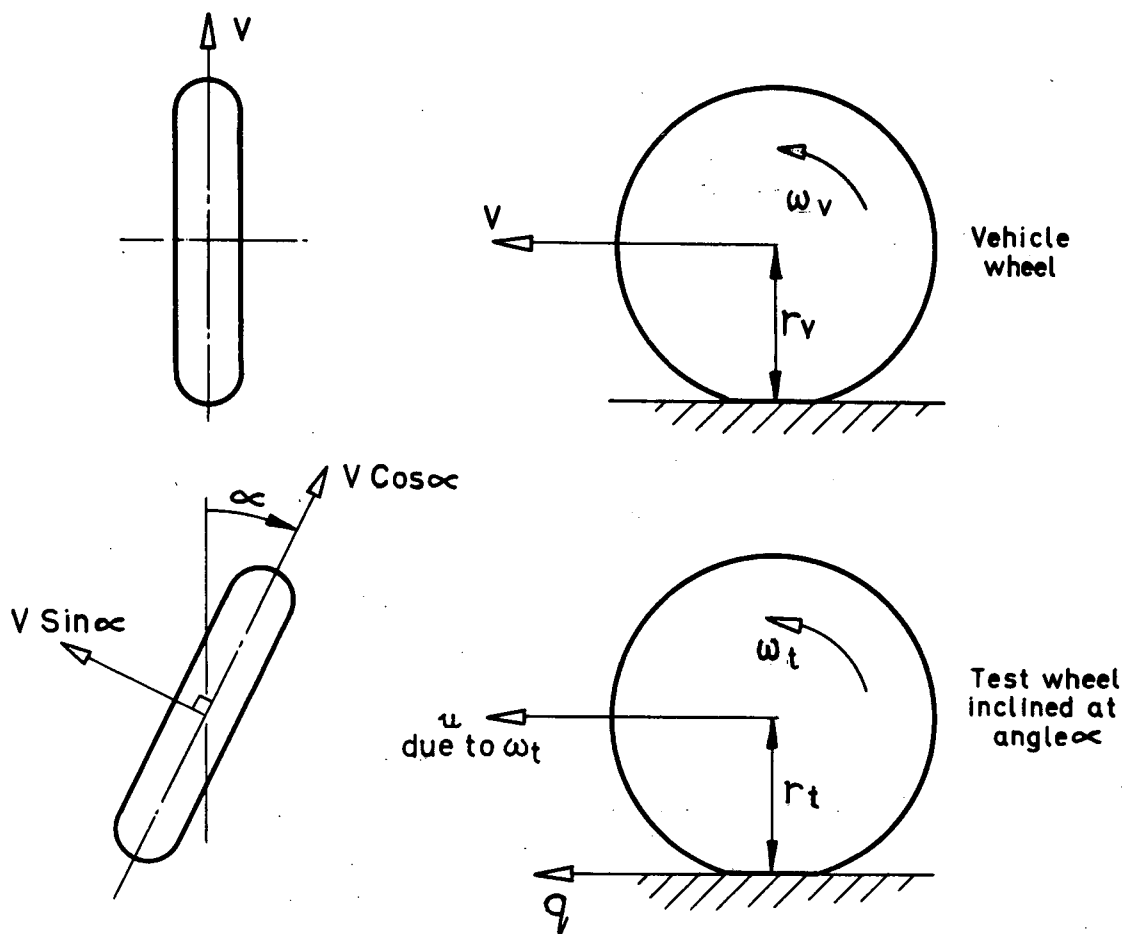


Fig. A3. LINEAR AND ANGULAR SPEEDS OF TEST WHEEL AND VEHICLE WHEEL

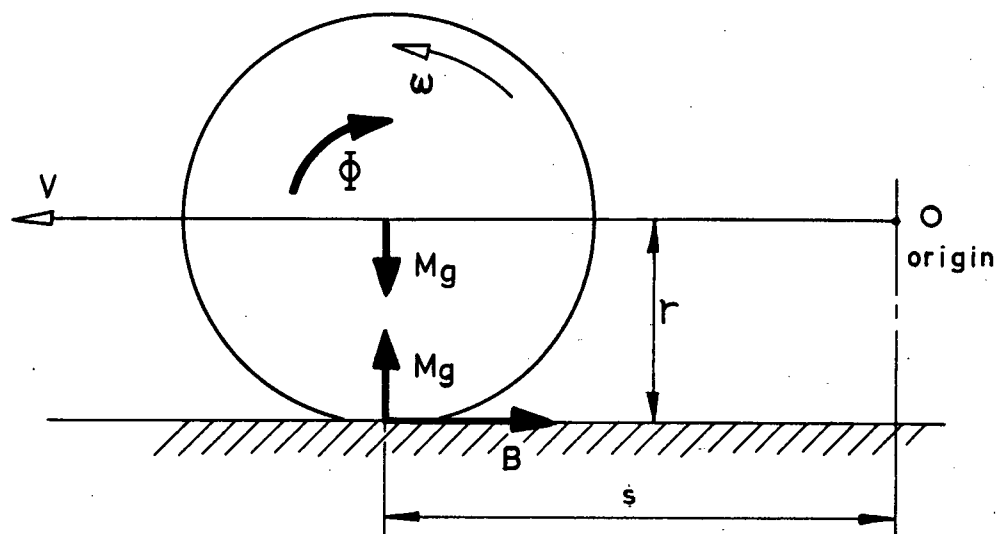


Fig.A4. THEORETICAL MODEL OF BRAKED WHEEL REPRESENTING A VEHICLE

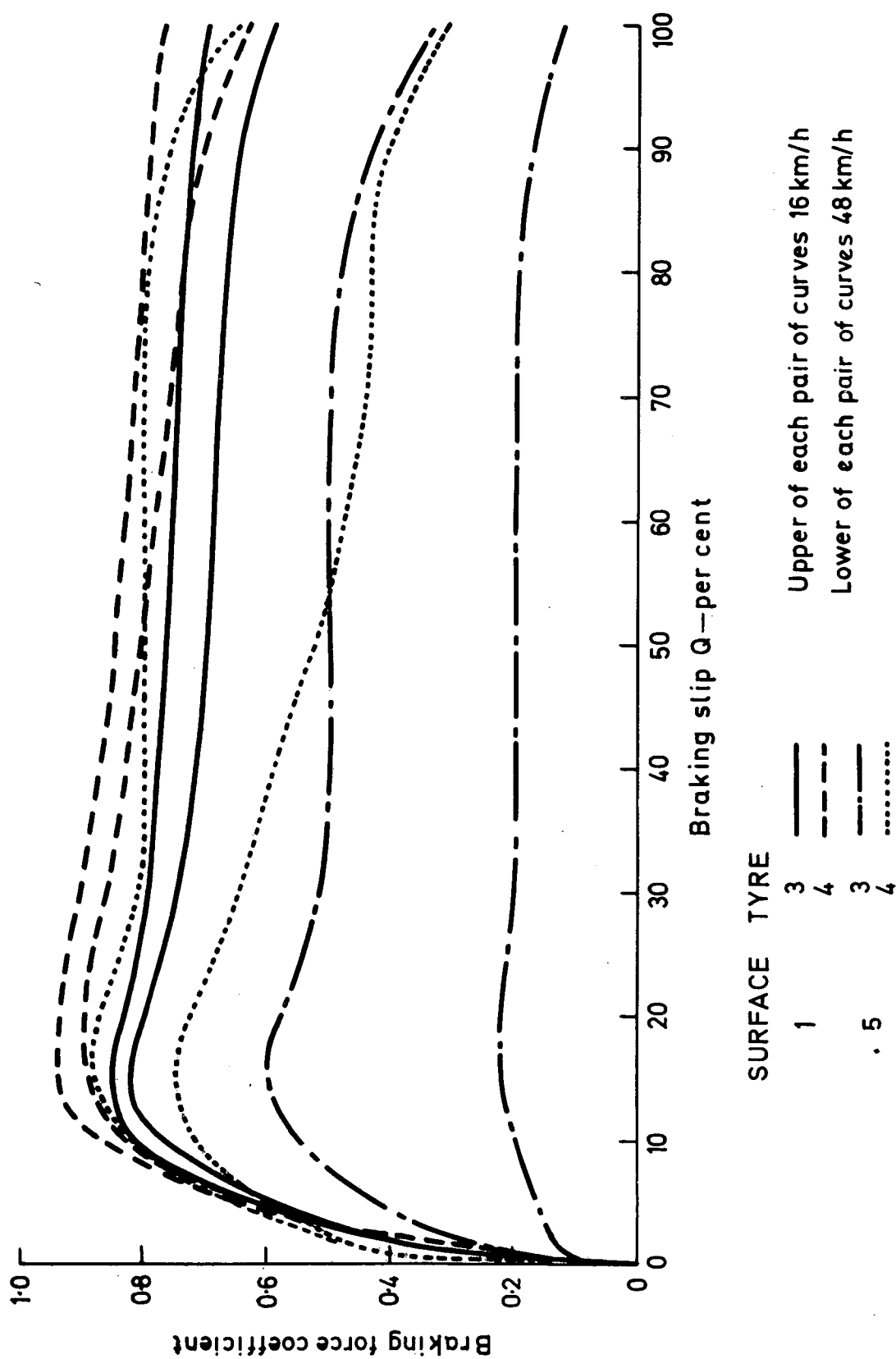


Fig.1. THE VARIETY OF CURVES PRODUCED WHEN BRAKING SLIP IS APPLIED BETWEEN FREE ROLLING AND LOCKED WHEEL CONDITIONS USING LOW RESILIENCE TYRES ON TWO SURFACES

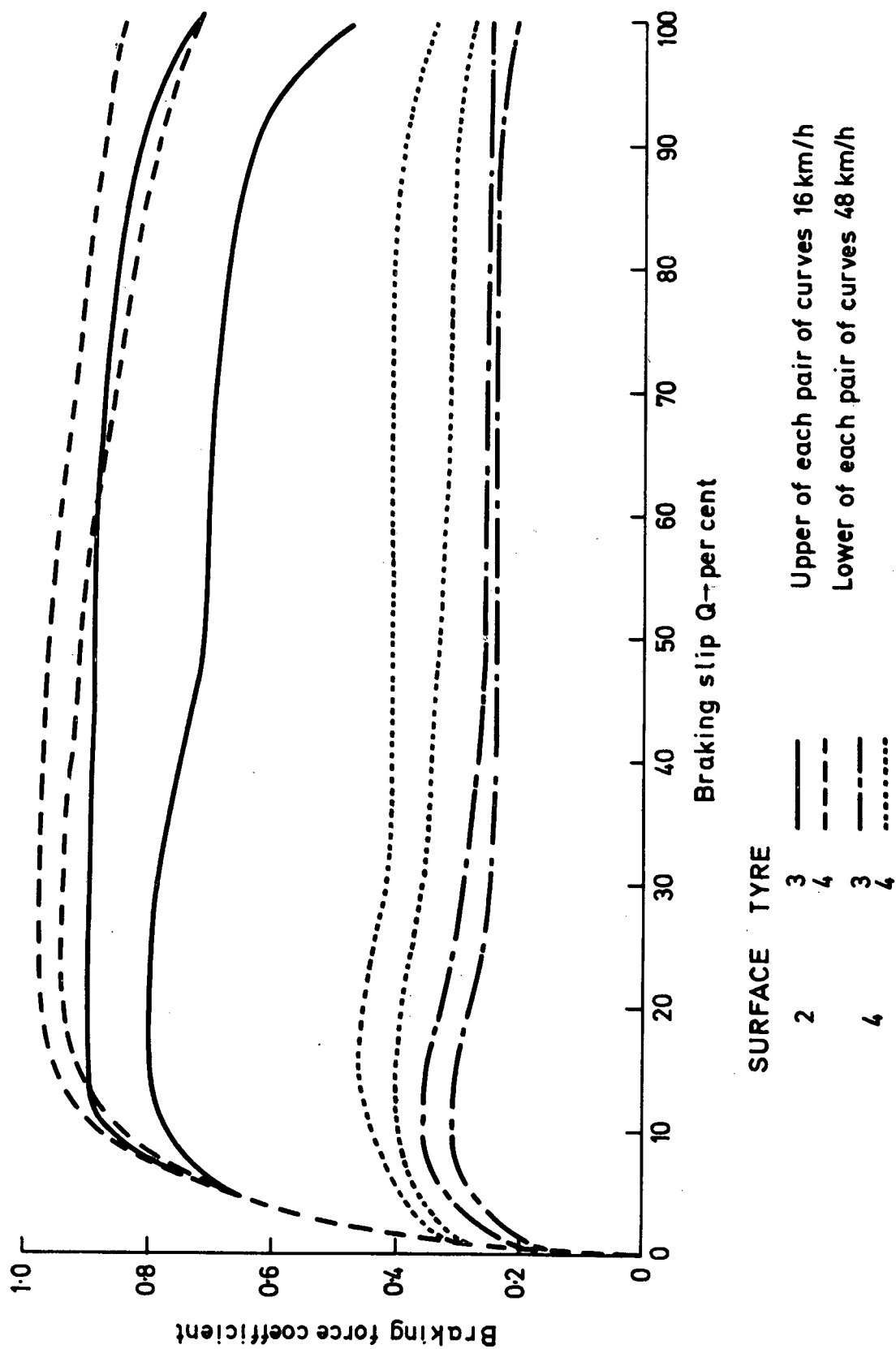


Fig.2. THE VARIETY OF CURVES PRODUCED WHEN BRAKING SLIP IS APPLIED BETWEEN FREE ROLLING AND LOCKED WHEEL CONDITIONS USING LOW RESILIENCE TYRES ON TWO SURFACES

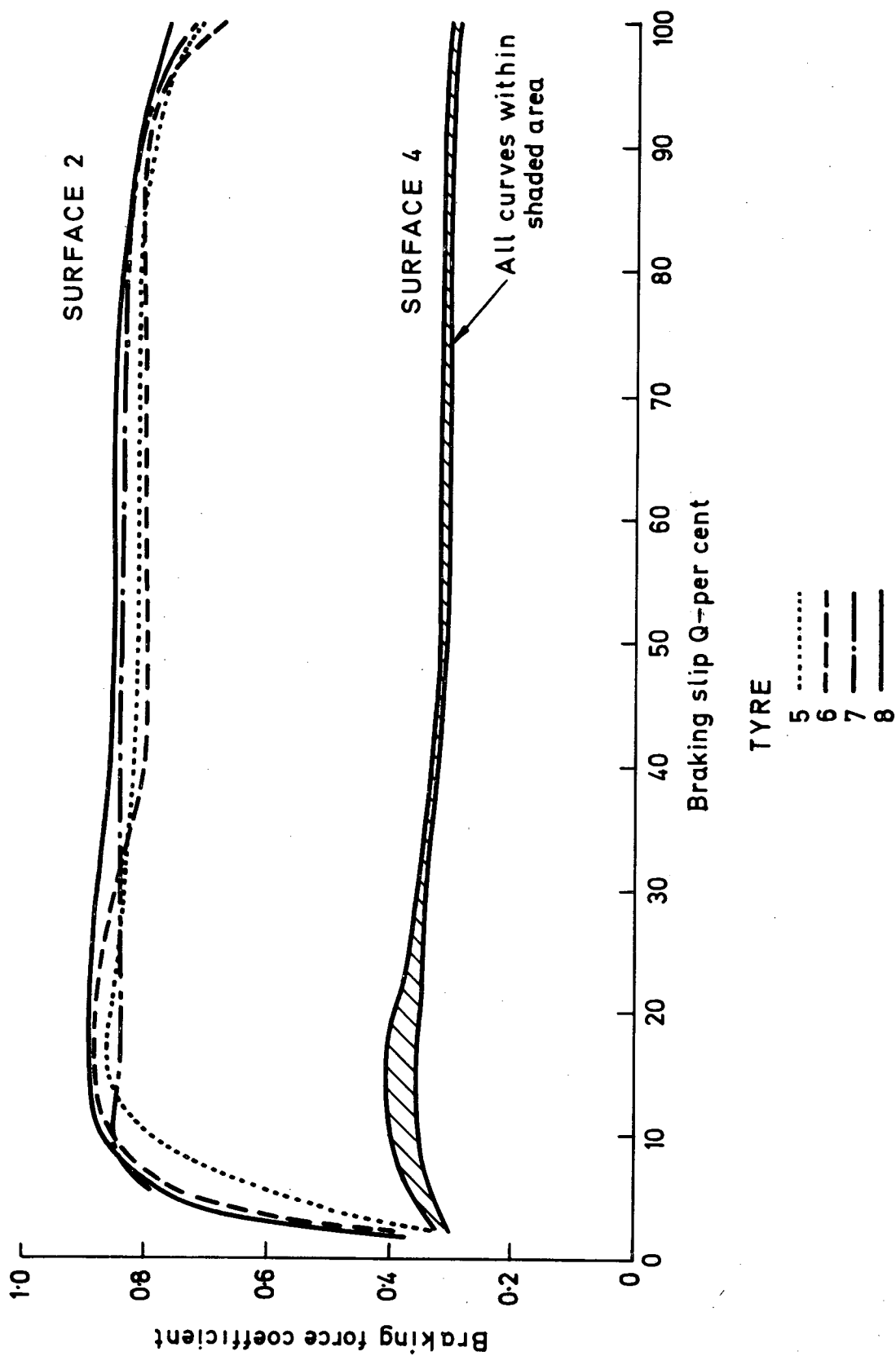


Fig.3. THE EFFECT OF CARCASS CONSTRUCTION

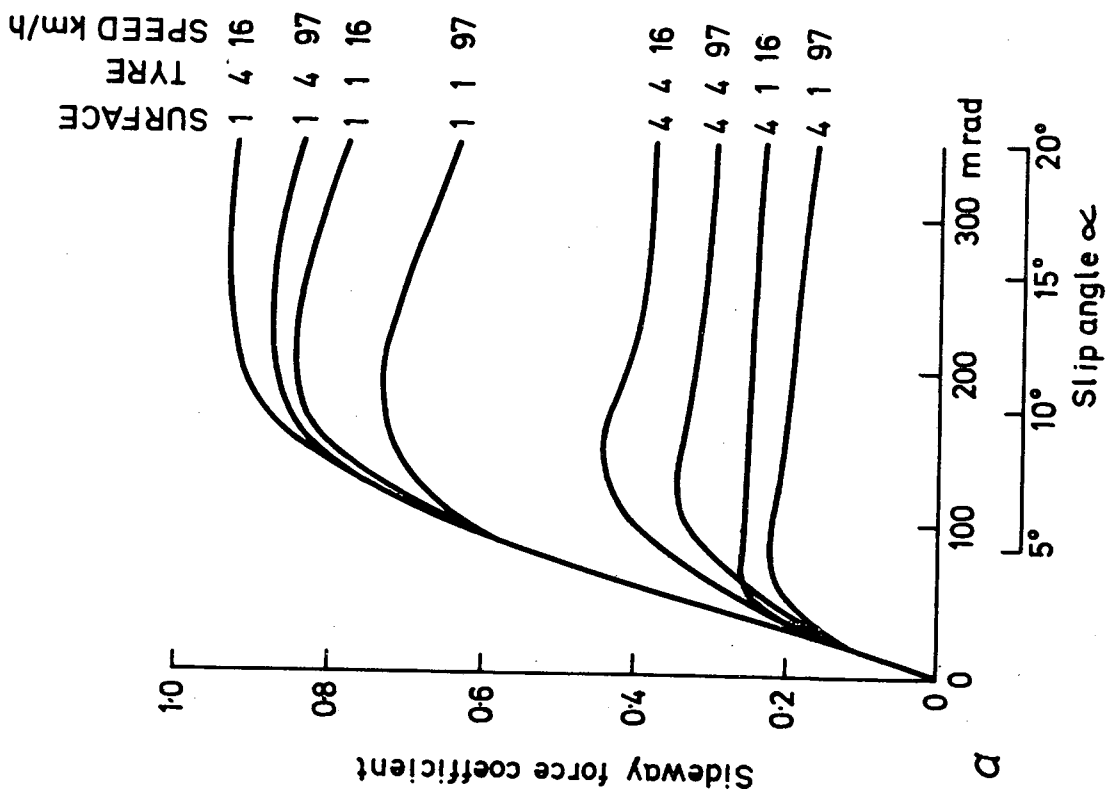


Fig.4a.

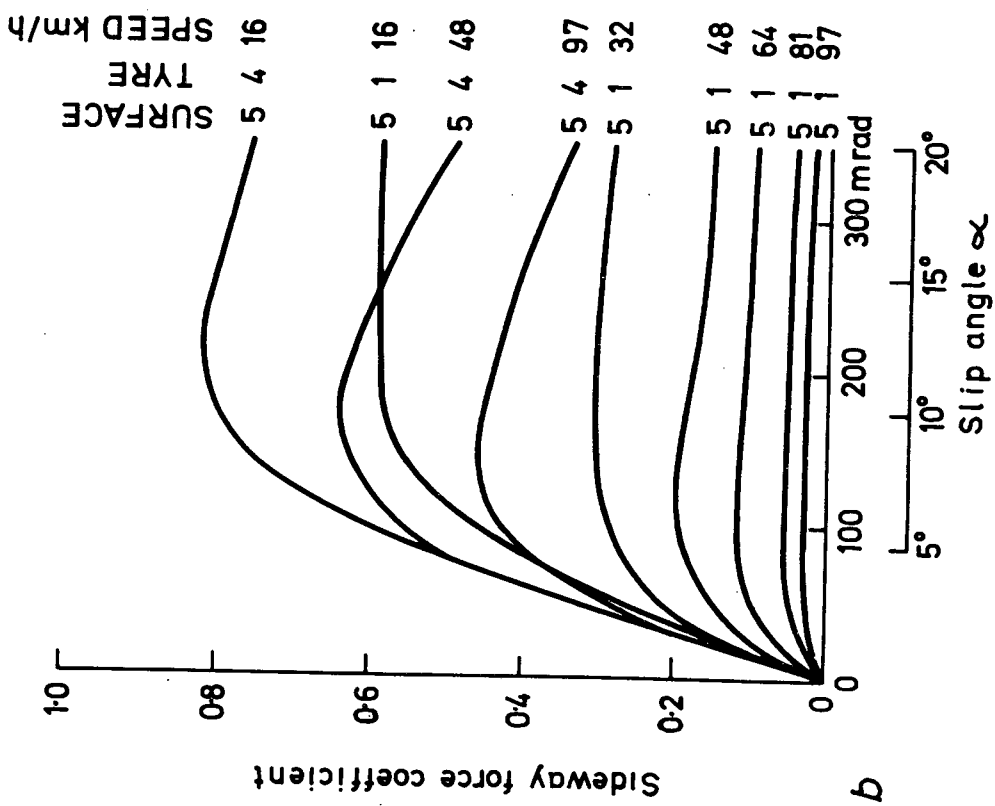


Fig.4b.

Fig.4 THE EFFECT OF SLIP ANGLE ON SIDEWAY FORCE COEFFICIENT

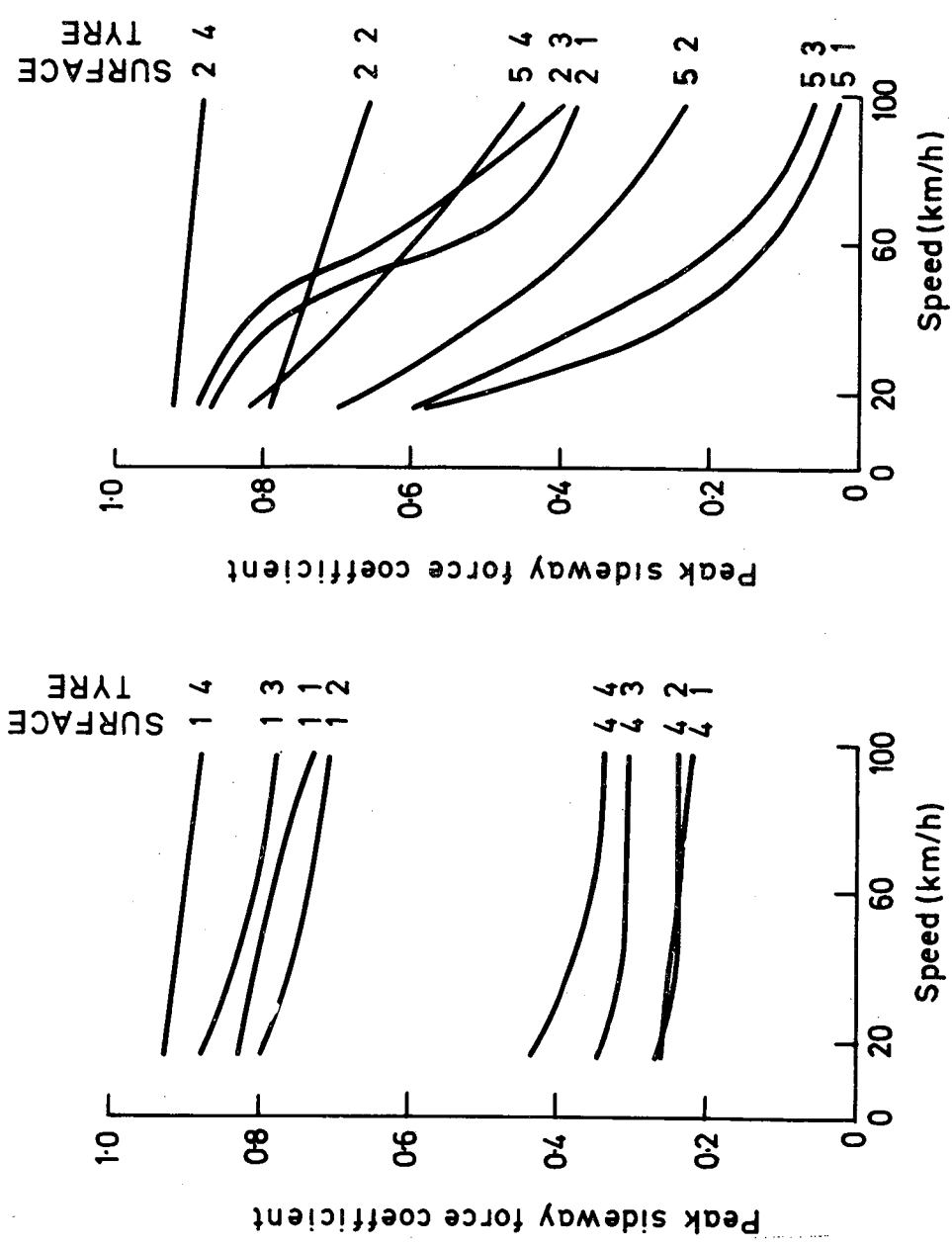


Fig.5. THE EFFECT OF SPEED ON THE PEAK SFC FOR DIFFERENT TYRES AND SURFACES

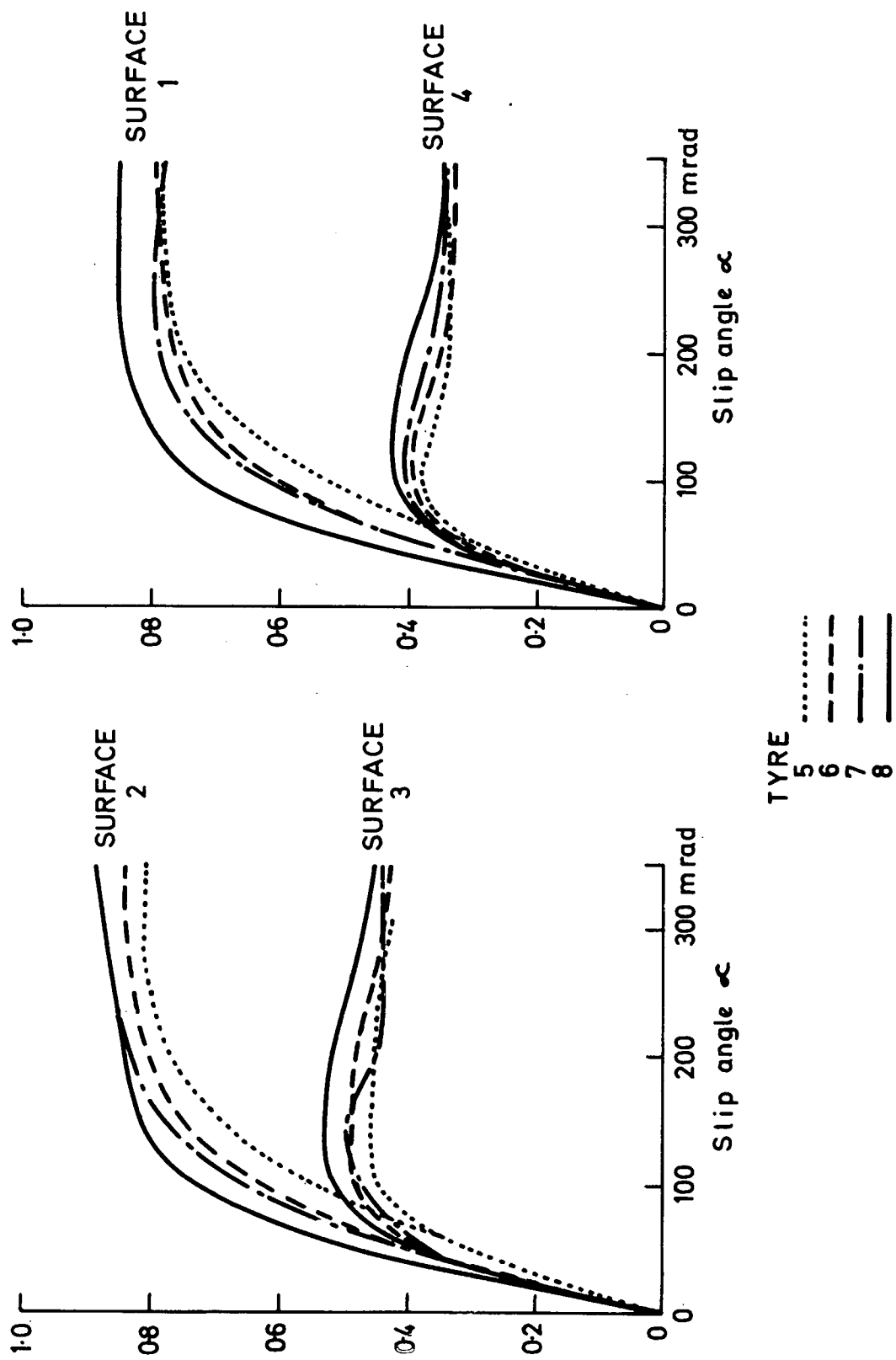


Fig.6. THE EFFECT OF CARCASS CONSTRUCTION ON THE DEVELOPMENT OF SFC

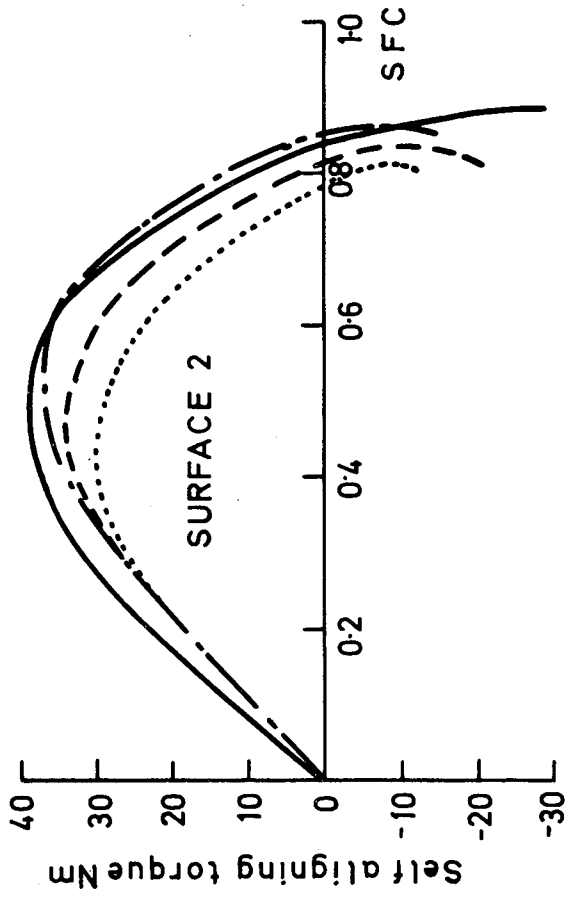
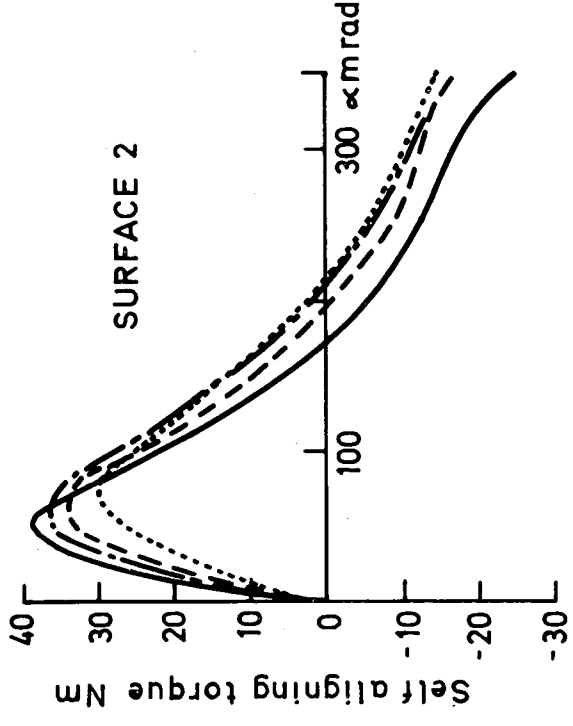
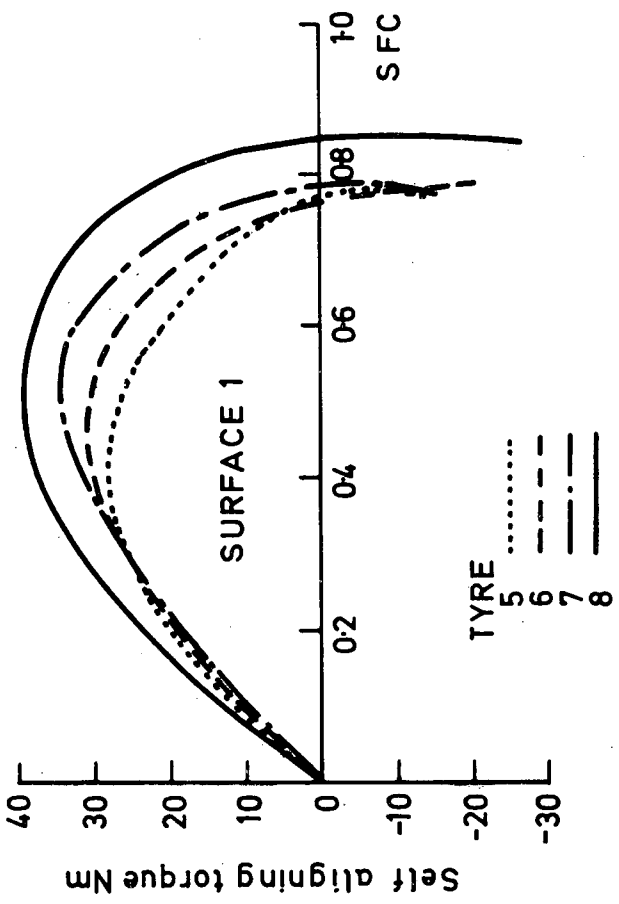
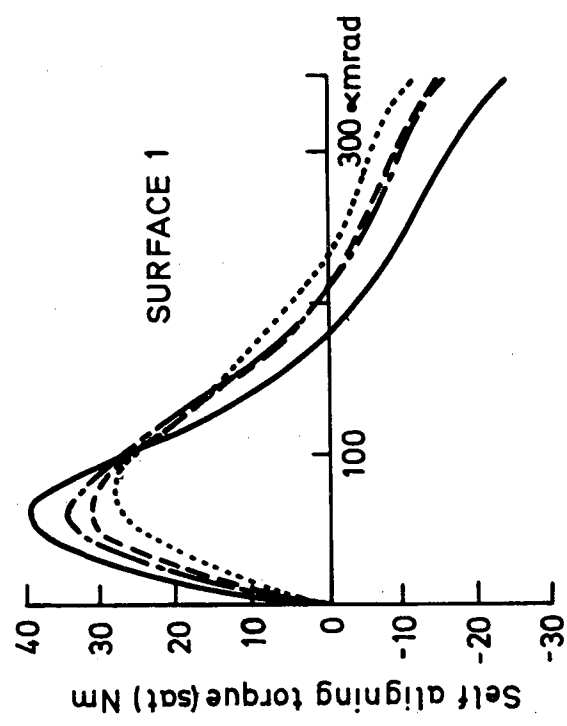


Fig.7 SELF ALIGNING TORQUE FOR PURE SIDEWAYS FORCE AND SLIP ANGLE

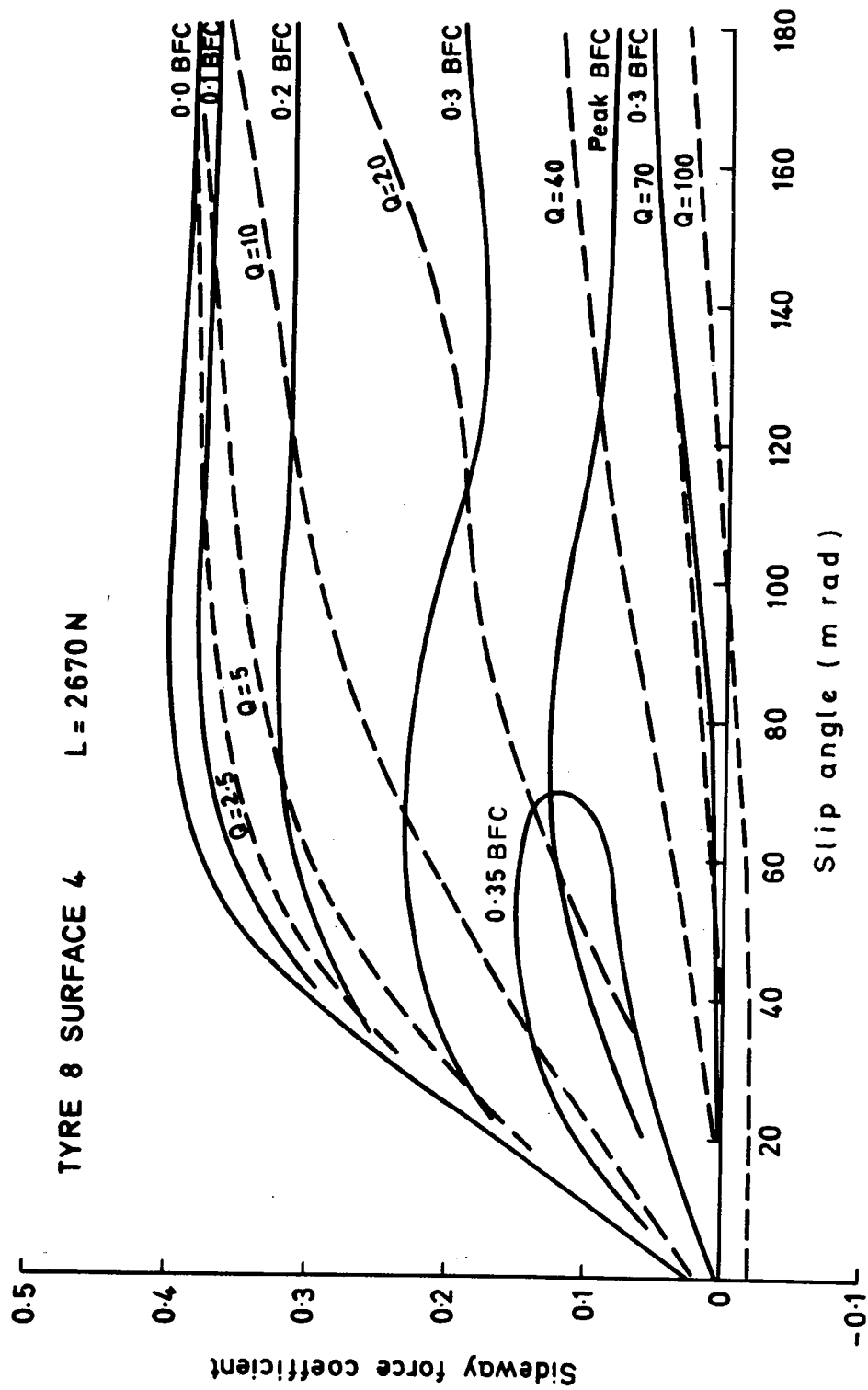


Fig. 8. THE INTERDEPENDANCE OF BFC AND SFC ON α AND Q

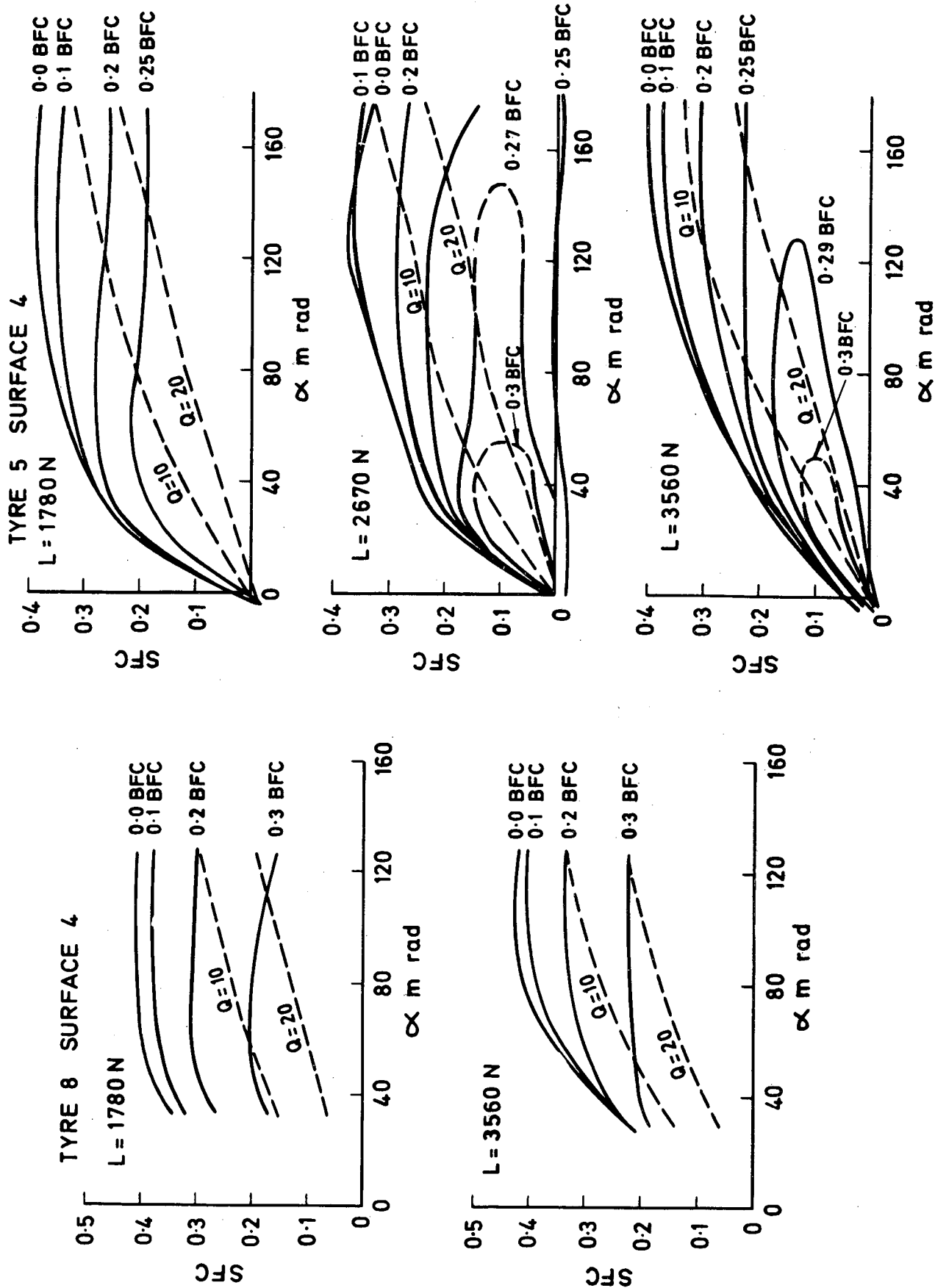


Fig. 9. THE INTERDEPENDANCE OF BFC AND SFC ON α AND Q FOR TYRES OF DIFFERING CARCASS CONSTRUCTION AND LOADING

SURFACE 4

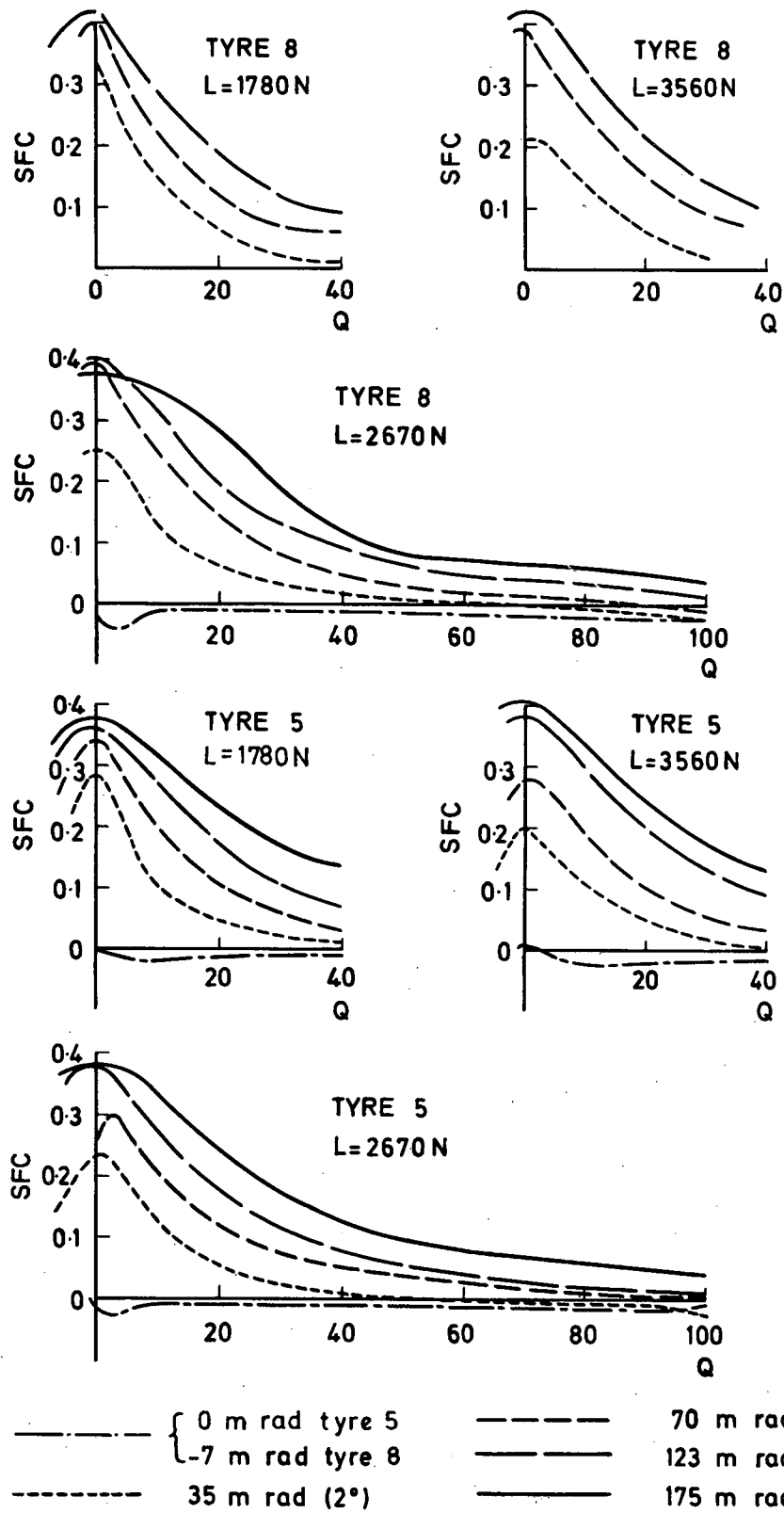


Fig. 10. THE EFFECT OF BRAKING SLIP (Q) ON THE SIDEWAY FORCE COEFFICIENT (SFC)

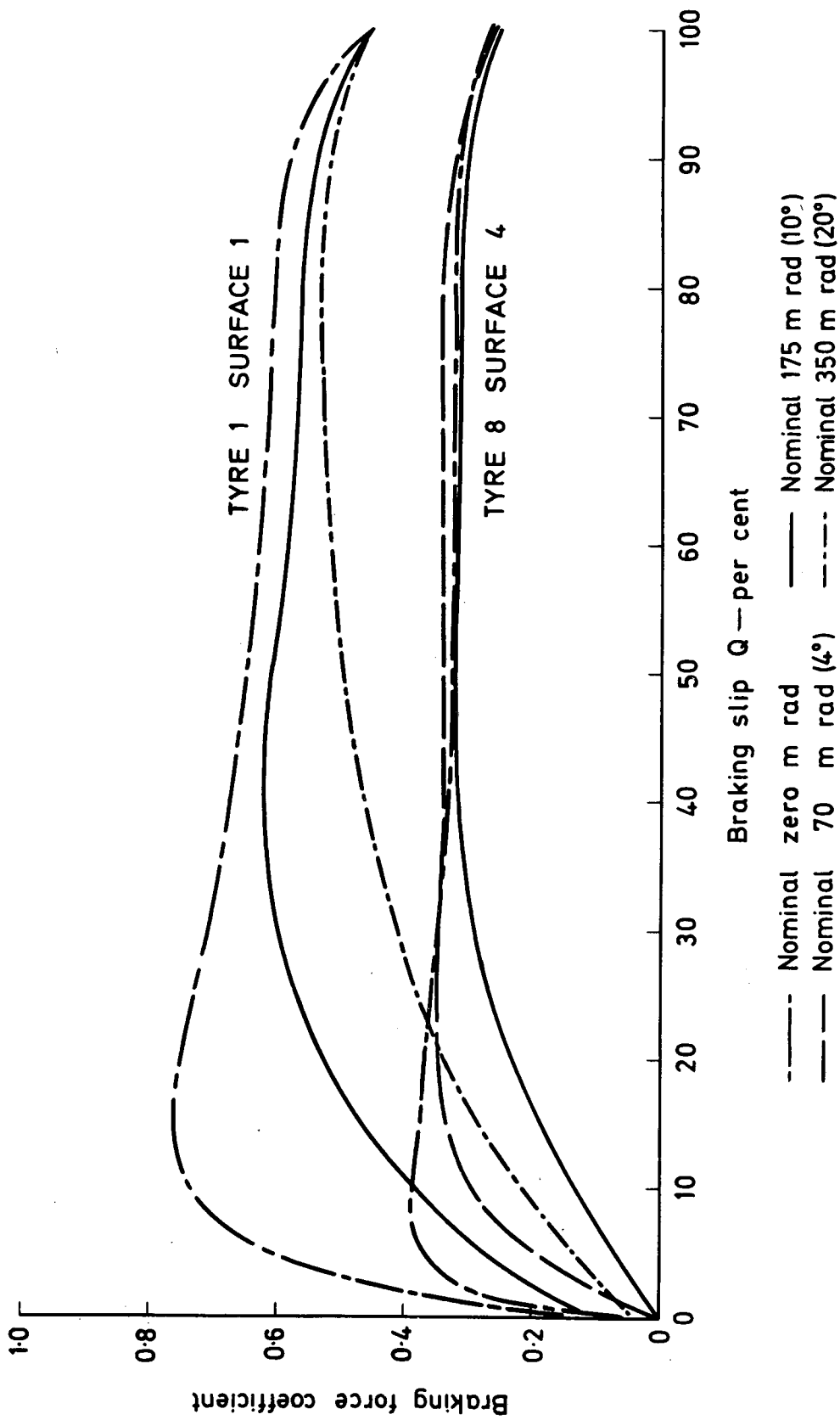
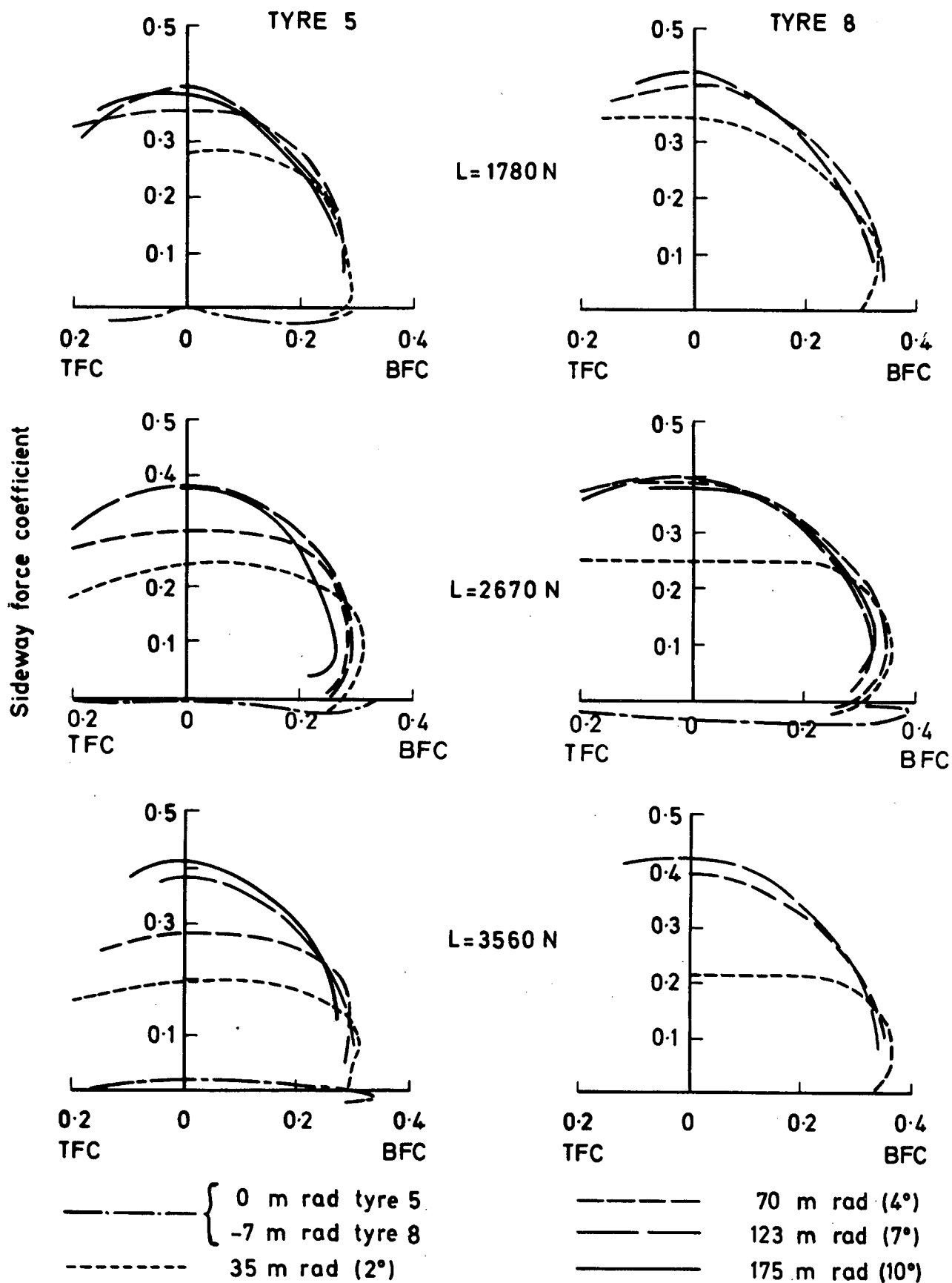


Fig. 11. THE EFFECT OF SLIP ANGLE ON THE BFC/Q RELATIONSHIP



TFC = Tracting force coefficient

Fig. 12. THE INTERACTION BETWEEN BRAKING & SIDEWAY FORCE COEFFICIENTS

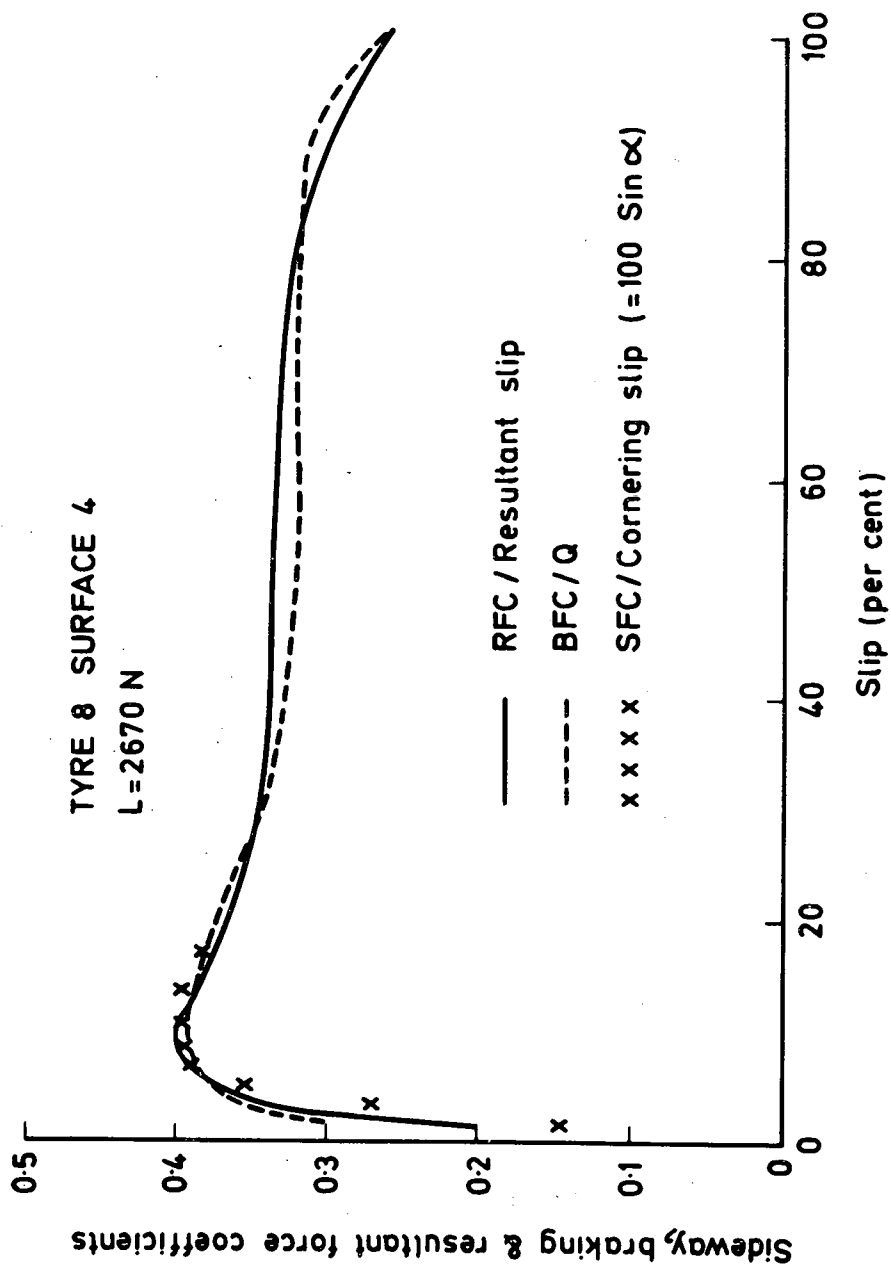


Fig.13. COEFFICIENTS PLOTTED AGAINST SLIP AT A FIXED VEHICLE SPEED

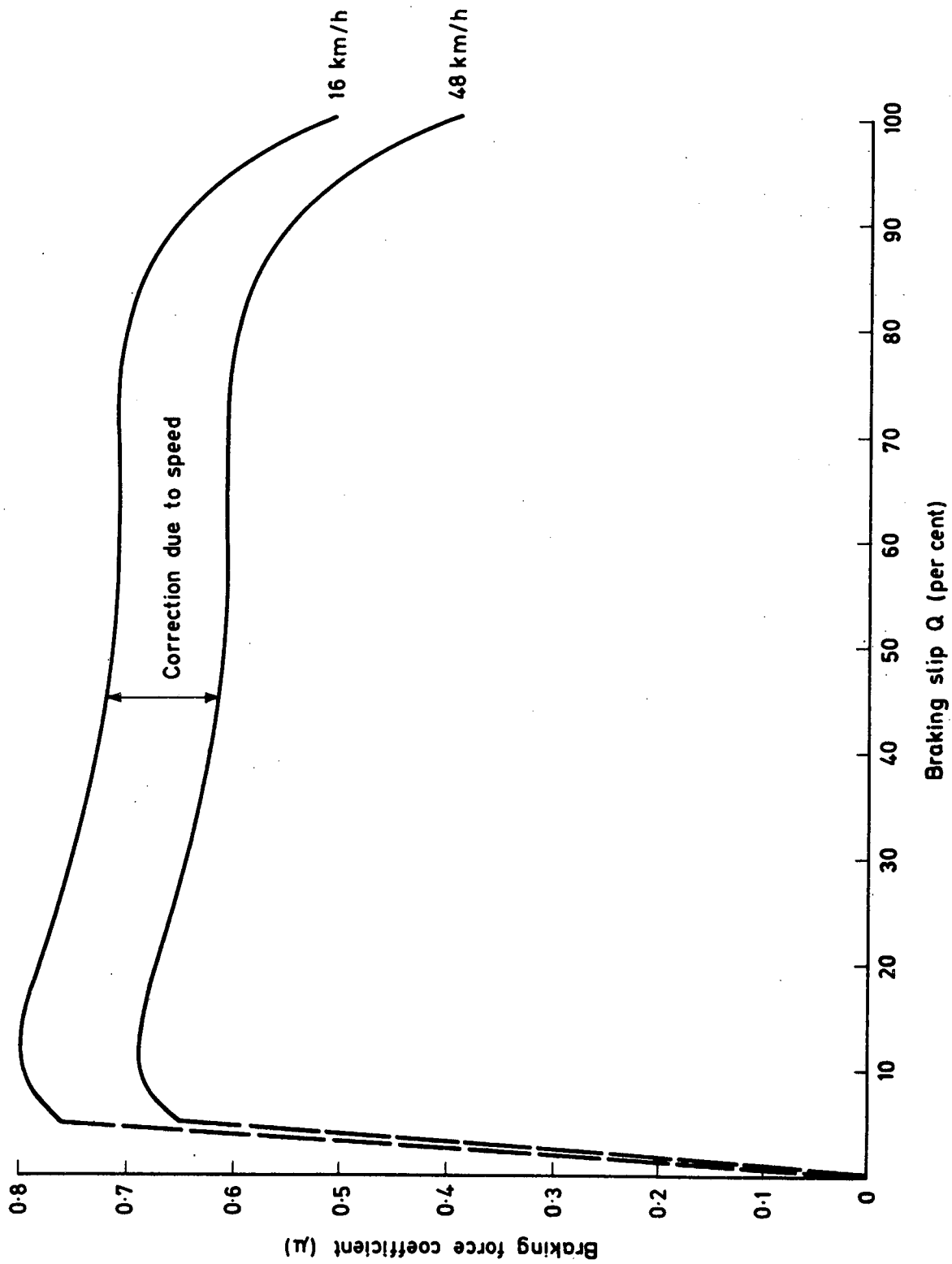


Fig.14. TYPICAL BRAKING FORCE COEFFICIENT/BRAKING SLIP CURVE AS USED IN THEORETICAL MODEL OF BRAKED WHEEL

ABSTRACT

Tyre forces as functions of cornering and braking slip on wet road surfaces: K. E. HOLMES and R. D. STONE B.Sc.: Ministry of Transport, RRL Report LR 254: Crowthorne, 1969 (Road Research Laboratory). The main purpose of this Report is to present and discuss empirical data on braking and sideway forces both separate and combined, and on self-aligning torque, all as functions of braking slip and slip angle. The data refers to wet road surfaces only, but the effect of such factors as speed, road surface texture, tread pattern, tread resilience, tyre construction and tyre wear are reported. The test vehicle recording system, data evaluation and analysis are also described.

It is suggested that a single relationship between the resultant force and the resultant slip, particularly if the true slip could be ascertained, might account for the larger part of the variations actually observed in the whole range of conditions from pure braking force through a combination of braking and sideway forces to pure sideway force. The importance of combined braking and cornering to the performance of antilocking brakes is shown. A theoretical study of the stopping distances implied by the braking force data under a variety of braking conditions, and statistical analyses of the pure braking and pure sideway force data are also included.