TRANSPORT and ROAD
RESEARCH LABORATORY

Department of the Environment
Department of Transport

TRRL LABORATORY REPORT 791

THE OPTIMISATION OF PUBLIC TRANSPORT IN SMALL TOWNS

by

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Crowthorne, Berkshire
1977
ISSN 0305—1293

CONTENTS

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ABSTRACT

A study was carried out with the aim of determining the optimum form of public transport for the town of Carterton, Oxfordshire. A demand model was developed and calibrated and was shown to be in close agreement with observations for three different levels of public transport in the town. This model was used to predict the variations of demand with fare and level of service for three different transport systems: fixed route minibus, many-to-many dial-a-bus and taxi. Supply models were developed for each of these types of operation, and the fare and level of service for each system was optimised according to both commercial and social objectives. It was concluded that none of these systems could be operated at a commercial profit and that only a fixed route minibus service could show a net social benefit.

1. INTRODUCTION

1.1 Background

Bus travel in the UK has been declining at an increasing rate over the last 25 years, reaching a peak of 4 to 5 per cent per annum during the 1960's. This decline has been arrested to some extent during recent years but this has been largely at the expense of increasing subsidies (reaching 20 per cent of total costs in 1975). This decline has primarily been caused by increased car ownership and rising costs which have resulted in increased fares and falling levels of service.

TRRL has been investigating the role of unconventional forms of service as an aid to stemming this decline by providing higher levels of service. The present report describes the comparative assessment of alternative forms of public transport in the town of Carterton in Oxfordshire.

1.2 Form of the analysis

The problem may be specified as the determination of which of a range of alternative transport systems is best suited to a small town. To gain understanding of these services, an intensive study was made of the way in which they would be used in the particular town of Carterton. Clearly it was not possible to experiment with each of the services operating at a range of fares and levels of service and so mathematical models were constructed of the demand for the services and of their operation. To carry out the analysis it was therefore necessary to:

1. decide the criterion to be used for judging the success of a given transport system,
2. specify which systems should be studied,
3. construct and calibrate the demand and operational models,
4. optimise each of the services by varying the fare, the fleet size and the capacity of the vehicles,
5. compare these optimal services to determine which of them best meets the specified objective.

These five stages in the analysis are considered in turn in Sections 2 to 6 of this Report. To simplify the presentation, the analytical details of the optimisation of each of the services are contained in Appendices 1 to 4. Each appendix comprises a description of one service and the method used to optimise the fare and level of supply of that service.

1.3 The study town

Carterton is situated 8 km from Witney and 23 km due west of Oxford and lies adjacent to Britain's largest RAF base at Brize Norton. The town is built around a crossroads at which are located the major social and shopping facilities. The population of 12,000 is spread fairly evenly over an area of approximately 3 km^2.

The town is served by half hourly bus services to Witney and Oxford and there are additional services to Swindon and Cheltenham. These services follow a circuitous route through the town but carry very few internal trips. A survey of the use made of these services was reported in Reference 1.

Because of the proximity of the RAF base, the population of the community is relatively young; the median age is 24 years as compared with 35 years for the entire population of the United Kingdom. In particular the middle aged and retired groups are under-represented. The transport systems considered in this study compete principally with walking and are therefore expected to be particularly attractive to members of these older age groups and to mothers with young children. It is therefore difficult to assess what effect this atypical age structure will have on the use of public transport.

2. CRITERIA FOR ASSESSING SERVICES

In order to determine the best way to operate a particular service or to assess which of two services is the better, it is necessary to determine the extent to which each type of operation achieves specified objectives. In the present work the transport systems have been assessed according to two criteria: profit maximisation and net social benefit maximisation.

2.1 Profit maximisation

This is a purely financial objective and is appropriate for use by the operator of the service. The operator's profit is, of course, the net revenue from the service, and its maximum is achieved at an equilibrium between increasing the revenue, by carrying more passengers and reducing the operating costs by reducing the capacity of the service.

2.2 Net benefit maximisation

Local and central government might be more interested in the social effects of the provision of transport which are measured by this objective. The net social benefit is measured by the benefit of the service to the community after subtracting the resource costs of providing the service. The only element of social benefit that is included in the present work is the saving in time gained by the users of the service. Other social benefits, such as the increased mobility of the aged or the benefits which do not accrue to users of the service, may be considered as additional benefits when deciding between different transport options in a particular town but at present, there is no generally accepted method of quantifying these effects.

3. THE ALTERNATIVE SERVICES

The transport systems studied were two fixed route minibus services, a dial-a-bus service and a taxi service. The two fixed route services have been operated in Carterton and were used to calibrate and validate the mathematical models. Details of the operation of the bus systems are given below.

3.1 The original minibus service

This service is described in Reference 1. It employed a single 15-seat minibus operating on four basic 15 minute tours, from which it would divert on request. Each tour covered one quadrant of the town with the minibus returning to its stand at the crossroads after each tour, so any point in the town had at least one service to and from the centre each hour. The service ran from 0915 to 1725 hours Monday to Saturday at a flat fare of 6p (children 3p) for each journey to or from the crossroads. A survey[1] carried out in June 1974 showed that approximately half of the patronage arose from one quadrant of the town. This imbalance in the demand resulted in a very inefficient service and, as reported in Appendix 1, had considerable influence on the optimisation of vehicle size.

3.2 The modified minibus service

In March 1975, as a result of the survey findings, the minibus routes were re-designed to provide a more frequent service to the areas of the town with highest demand for travel. At the same time the base fare was increased to 10p, with a 15p fare for journeys from one quadrant of the town to another.

3.3 The dial-a-bus service

The general ('many-to-many') form of this service consists of a fleet of buses in radio communication with a control centre. A customer requests service by telephoning the central control giving his location and the destination to which he wishes to be taken. This information is combined with information describing the vehicle positions, tentative routes, and trip characteristics of other passengers and a vehicle is assigned to serve the customer. He is then advised when to expect the bus to arrive. Other passengers may be picked-up or set-down before he reaches his destination and his route is therefore not direct. In this way, the dial-a-bus operates exactly like a shared taxi and consequently has higher productivity than an ordinary taxi. It is therefore possible to charge lower fares in return for the lower level of service being provided in terms of longer waiting and riding times.

In addition to catering for telephone demands for immediate service, dial-a-bus systems are usually designed to provide a hail-stop service and to enable trips to be booked in advance on a regular basis. However, the destinations served are often restricted to shopping centres, railway stations or other major trip attractors. Such a system provides a restricted service which is intermediate between that of a fixed route minibus and that of a shared taxi. In the present study the dial-a-bus service is taken to provide transport between any two points in the town.

The supply model for this service was not sufficiently sensitive to permit optimisation of the vehicle size. However it was possible to determine whether four seat vehicles would have sufficient capacity to operate the service or whether it would be necessary to employ minibuses. In differentiating between these two types of operation they are referred to as shared taxi and dial-a-bus services, respectively. These services are not distinct, in that they have identical modes of operation. However, their analysis differed not only in the vehicle size but also in the costing which was used. It was assumed that a shared taxi service would be provided by a private operator and would not be eligible for the new bus grant or fuel duty rebate that a public transport undertaking would receive when operating a dial-a-bus service.

With the exception of the shared taxi system, all of the types of service considered here are already operating in this country and their characteristics are well-known. A shared taxi service cannot be operated under existing legislation and so less is known about its practical aspects: but with this limitation it is of research interest to study the relative merits of such a service. (It is interesting to note that a dial-a-bus service employing 4 seat vehicles could be operated in this country provided that road service and public service vehicle licence were obtained*. The service would then be eligible for new bus grant and fuel duty rebate).

All of the services were taken to operate only during the daytime off-peak period (0915 to 1730 hours). It is believed that extension of the operating hours into the peaks or the evening would not change the ranking of the services but, by making use of available resources, might improve their profitability.

4. THE STRUCTURE AND CALIBRATION OF THE MODELS

4.1 The demand model

The demand for each of the transport systems was estimated by using an exponential modal split model[2] and an allowance was made for the number of trips which the service itself would generate. The model distributes trips between any number of transport modes according to the fare, riding time, waiting time, access time and egress time for each mode.

The procedure for calibrating the model consisted of:

1. dividing the town into zones,
2. calculating the fares and the components of travel time for journeys made on each mode of transport between each pair of zones,
3. carrying out a travel survey to determine the number of trips made on each mode between each pair of zones,

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* The Post Office have successfully licensed passenger cars to operate their postbus services.

4. applying the demand model with the measured fares and travel times to predict the number of trips which would be carried by each of the modes between each pair of zones,

5. adjusting parameters in the model until the computed trip matrix corresponded as closely as possible with the observed matrix.

In the present case, not only was a household travel survey[3] available but detailed information was also known about the trips that were made on the original minibus service[1]. This information included details of the mode of travel which would have been used if the minibus service had not been operating. It was therefore possible to construct the travel matrix which would have been observed in the absence of the minibus service. Thus trip matrices were available for the situations both before and after the introduction of the minibus service and the adjustable parameters in the demand model were chosen so that the computed trip matrices agreed as closely as possible with *both* of these situations. However, it was found that the parameters assumed very similar values when the model was calibrated using each of these sets of data separately. It follows that the calibration based on one of these situations gave a model which was capable of accurately predicting what would happen in the other.

As was done in the study described in Reference 2, a large number of test calibrations were made using different adjustable parameters and, in the present case, it was concluded that two parameters only should be varied. These were the car availability (as defined in Reference 2) and the sensitivity of the travellers to changes in the cost of travel. The values of time were held constant at the values normally used in DTp assessments, these were 28 pence per hour for in-vehicle time and 56 pence per hour for travel time not spent in a mechanical mode. The observed and calculated total numbers of trips made on each mode before and after the introduction of the minibus, are shown in Table 1 and an analysis of the trips carried by the minibus service is presented in Table 2.

TABLE 1

Comparison of the observed and calculated number of trips made on each mode
(percentages are shown in parenthesis)

Mode	Trips per weekday made between 0915 and 1730 hours			
	Before		After	
	Observed	Predicted	Observed	Predicted
Car and motorcycle	3961 (33.8)	3673 (31.4)	3958 (33.8)	3656 (31.2)
Walk and bicycle	7708 (65.8)	7979 (68.1)	7630 (65.1)	7897 (67.4)
Stage bus	46 (0.4)	65 (0.6)	41 (0.4)	62 (0.5)
Taxi	2 (0.0)	0 (0.0)	0 (0.0)	0 (0.0)
Original minibus	–	–	96 (0.8)	109 (0.9)

TABLE 2

Trips carried by the original minibus service

Mode foregone	Trips per Day	
	Observed	Calculated
Car and motorcycle	3	17
Walk and bicycle	78	82
Stage bus	5	3
Taxi	2	0
Generated	7	8
Total	96	109

The modal split model was not capable of predicting the number of trips which would be generated by the minibus service and the prediction for generation shown in Table 2 was obtained by assuming that the generated trips form a constant proportion of the total trips on the mode. This was taken to be 7.1 per cent, as observed on this service, and was held constant for all subsequent predictions. Bearing in mind that the minibus service carried only 0.8 per cent of all trips and that the figures in Table 2 were obtained by differencing pairs of large numbers, the calculations are felt to be in good agreement with the observations. The major discrepancy was that the model diverted too many trips from car onto the minibus service. It was found that this could only be corrected by introducing a third parameter and it was felt that the data were not sufficiently accurate to justify this.

The calibration of the demand model was validated by comparing the predicted and observed patronage of the service when it was modified (395 and 385 passengers per week, respectively). Further confirmation of the accuracy of the demand model was provided by its close agreement with the observed geographical distribution of the demand for the two minibus services.

It is concluded that the calibrated demand model produced results which were in close agreement with the three different situations of (a) no internal transport service, (b) the original minibus service at a fare of 6p and (c) the modified minibus service at a higher fare.

4.2 The supply models

When used predictively, the demand model describes how the demand for a particular mode varies with the fare that is charged and with the different elements of the travel time. That is, the demand model relates the demand, as a dependent variable, to the service characteristics, as specified by the independent variables of fare, waiting time, riding time etc. However, in most transport systems the components of travel time are themselves a function of the demand. For example, if the fleet size is held constant the waiting time for a taxi increases with the number of people using the service. In any transport system therefore there is a balance, for a given level of supply, between the demand and the quality of service to the passenger. If, in the taxi system, one considers a small displacement from this equilibrium by, say, increasing the demand, the increases waiting time that would result would prove unacceptable to some of the travellers and so the demand and waiting time would fall to their equilibrium values.

This relationship may be shown mathematically by the pair of equations:

$$d = F(w)$$
$$w = G(d)$$

The first of these equations represents the demand model and shows the demand, d, to be a function of the waiting time, w. The second represents the supply model in which the waiting time is a function of the demand. The equilibrium between the supply and demand is obtained by simultaneously solving these two equations for the equilibrium values of the demand, d_0, and the waiting time, w_0.

As described below, each type of service required a different supply model. The details of the calibration and application of these models are given in Appendices 1 to 4.

4.2.1 The supply model for fixed route services For a fixed route service the time that a passenger spends riding in the vehicle increases only slowly with demand. This increase principally arises from the boarding time of the additional passengers. For the range of demand considered in the present work this increase is not significant and the riding time was taken to be independent of demand. Similarly, increase in the demand has little effect on the time spent waiting for a bus until the demand reaches the saturation level of the service. At this point the buses start running full and the passenger waiting time may be suddenly increased by the headway of the service (the time between two successive buses). For the short journeys and long headways being considered, it is unlikely that, in these circumstances, any passengers would wait for the following bus; they would walk instead. This situation is best reproduced in the modelling by the use of a capacity limit. This involves specifying a given waiting time and estimating the resulting demand and its distribution over the bus tours that are operated. If the demand for any tour exceeds the vehicle capacity the number of passengers carried on that tour is set equal to the vehicle capacity. Thus, on some tours the entire demand is satisfied whilst on others the vehicle capacity determines the number of passengers carried. With this procedure there is no direct feedback from the demand to the waiting time experienced by those passengers who are actually carried. For instance, if a waiting time of ten minutes is specified and the resulting demand exceeds the capacity of the service, it is assumed that the passengers who are actually carried on the service wait for ten minutes while the remainder of the demand is forced into some alternative mode. This is equivalent to assuming a discontinuity in the waiting time whereby it equals ten minutes for all trips up to the capacity of the service and infinity for any further trips.

In order to carry out this analysis it is necessary to determine the way in which the demand is distributed over the bus tours. In Appendices 1 and 2 this is taken as the observed distribution for each of the two fixed route services, and the distributions themselves are assumed to be independent of demand. That is, a percentage increase in the demand results from the same percentage increase in the demand for each bus tour.

4.2.2 The supply model for a dial-a-bus service For this type of service any increase in the demand increases both the number of route diversions and the number of stops that the vehicles make. The passenger waiting and riding times therefore increase progressively with the demand. Thus, at very low demand a person requesting service is quickly served and may be carried directly to his destination. At a higher demand, the pick-up is inserted into a vehicle route and may not be made until after a number of other pick-ups or set-downs. This increases the passengers' waiting time. Also he will be carried to his destination via the points where other passengers board or alight from the bus and he therefore experiences a longer riding time.

The mean journey time by dial-a-bus may be related to the direct journey time (by car) in the form

$$\left\{\begin{matrix}\text{mean dial-a-bus}\\ \text{journey time}\end{matrix}\right\} = (1+x)\left\{\begin{matrix}\text{mean car}\\ \text{journey time}\end{matrix}\right\}$$

where the variable x increases with the size of the service area and with the demand density but decreases with the number of vehicles in the dial-a-bus fleet. On curve fitting the results of extensive computer simulation[4,5] x has been shown to have the form

$$x = \left\{\frac{A(0.82 + 0.087D)}{n}\right\}^2$$

where the journey times are measured in minutes. In this expression A is the size of the service area, in square miles; D is the demand density rate, measured in trips per square mile per hour, and n is the number of vehicles in service. The simulations on which this equation is based were for larger service areas employing more vehicles than in the present work. It is therefore possible that the equation is being used beyond its range of applicability. However, the only alternative to its use would be to repeat the simulation for the cases of interest. Since the equation does give plausible waiting and riding times when applied to Carterton, it was felt that such extensive computation was not justified.

The foregoing analysis is concerned with how quickly a dial-a-bus can respond to a request for service, and the way in which this response time varies with the demand. There is therefore the implicit assumption that the number of trips carried is constrained only by the willingness of the passengers to wait for service, and that the service is capable of carrying the demand which is generated by its response time. In principle, the capacity could be optimised by a method similar to that used for a fixed route service. Thus, given the demand and its distribution in time together with the distribution of passenger riding times, it would be possible to calculate the probability that the vehicle would be required to carry more than a specified number of passengers at one time. From these probabilities it would then be possible to determine the proportion of trips which would be supressed by operating a vehicle with a given number of seats. However, insufficient is known about the operation of a dial-a-bus service to permit such an accurate optimisation of vehicle size. Therefore only a coarse analysis has been made of whether the service could be operated as a shared taxi, using four-seat vehicles, or whether it would require a minibus.

The details of the application of this supply model are given in Appendix 3.

4.2.3 The supply model for a taxi service In this case the passenger riding time is independent of demand and, because each vehicle serves only one journey at a time, the waiting time may be obtained by the application of queueing theory. Thus, when a passenger telephones for service he joins a queue and waits until all the passengers preceding him in the queue have been served. When he reaches the head of the queue he is served by the first vehicle which is free. His service time is the time taken for the vehicle, once free, to reach his origin and then take him to his destination, at which point the vehicle again becomes free.*

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* It should be noted that the time between requesting service and being collected (the waiting time) is longer than the queueing time, which is the time between the request for service and the assignment of a vehicle.

This problem is only exactly soluble for the case of one vehicle with demand distributed randomly in time, but an approximate solution is possible for the more general case[6] and is described in Appendix 4.

4.3 The benefit model

The social benefits associated with the introduction of a local public transport service may be divided into four categories:

1. the time saving benefits to the users of the service,
2. the savings in resource costs associated with the reduction in vehicle mileage by diversion of trips from other mechanical modes,
3. non-user benefits, such as reduced road congestion,
4. other unquantifiable social benefits, such as the increased mobility of the aged.

Of these four types of benefits, only the first two were included in the present work. There were found to be no significant benefits to those who did not use the service and those benefits falling into the fourth category were omitted, for their inclusion would have involved value judgements inappropriate in the present work. It was found that the savings in resource costs never exceeded three per cent of the time saving benefits.

The benefits resulting from savings in passenger time were calculated directly from the demand model, as described in Reference 2, according to the following scheme:

1. the consumer surplus (the additional amount that the passengers would be willing to pay to retain the service) was obtained from the demand model,*
2. from this was subtracted any money saving that the passengers made on transferring to the new mode,
3. the remaining quantity is the passengers' valuation of their time savings and, since the behavioural value of time was taken to be the same as the equity value, this is also the social value of the time savings.

5. OPTIMISATION OF THE INDIVIDUAL SERVICES

This is the second stage in the modelling process. As described in the previous section, the first stage consists of determining the equilibrium that exists between the supply of a transport system and the demand for it. For example, it may be decided to provide a system of two taxis at a fare of 15p. At the equilibrium between demand and supply it would be found that this system would carry about 660 passengers per week with a mean waiting of nearly 9 minutes.

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* The determination of the consumer surplus in a situation where the level of service is a function of the demand is discussed in Appendix 6.

The stage of optimisation involves selecting from the set of all such equilibria that one which best achieves the specified objective. Thus, in the taxi example quoted above, the fare may be optimised by repeating the calculation at different fares. It would then be possible to determine the fare which maximises the revenue; this is the profit maximising fare for a fleet of two taxis. The fare optimisation may then be repeated for a fleet of, say, three vehicles. If the operating costs are known, these two optimal solutions may then be compared to determine which fleet size may be most profitably operated. For the case of a fixed route service this process involved a third stage at which the vehicle size is optimised.

To carry out this optimisation it was necessary to know the operating costs and the way in which they vary with fleet and vehicle size. The resource costs of different operations were similarly required in order to determine the services which maximised the net social benefit. An analysis of these costs, in June 1974 prices, is presented in Appendix 5.

6. COMPARISON OF THE SERVICES

The final stage of the analysis is a comparison of the relative merits of the alternative services when each is operated at its optimum fare, vehicle size and fleet size. The characteristics of these optimal operations, for both profit and benefit maximising services, are shown in Table 3. On comparing the two sets of services, it may be seen that to maximise net benefit it was necessary to attract more passengers than were required for profit maximisation. This required operating the service at a lower fare and with more capacity. Once this condition had been fulfilled, the relative merits of the alternative systems were found to be the same whichever of the two objectives they were designed to satisfy.

Considering first the profit maximising operations, it was found that none of the systems considered could be operated at a profit. This is a consequence of the fact that internal transport services in such a small town compete directly with walking. Thus, although there are a large number of internal trips made in the town, they cannot be attracted on to a mechanical mode at a sufficiently high fare to recover the cost of operating a service. It was found that the least loss would be made by the service having the lowest operating costs, that is the fixed route minibus service.

It is, however, of value to analyse the relative performance of the different services to assess their commercial viability in other settings. The profitability of each of the systems is determined by its revenue earning capability and by its operating costs. The revenue, in turn, is determined by the number of passengers which can be attracted on to the service at a particular fare and by the capacity of the service to carry those passengers. It was found that the number of passengers per vehicle hour (the vehicle productivity) fell rapidly as the level of service was improved. Thus, the best fixed route service showed three times the vehicle productivity of the dial-a-bus service which, in turn, had twice the productivity of a taxi service. However, the high level of service offered by the demand responsive systems proved capable of attracting far more passengers than were attracted to the fixed route services at the same fare. Since they did not have the capacity to carry these passengers, it was possible to limit the demand on the taxi and dial-a-bus services by charging higher fares. The change in revenue on improving the service was therefore determined by whether the improvement would allow a sufficient increase in fare to off-set the reduction in productivity. It may be seen from Table 3 that, by changing from the modified minibus service to the dial-a-bus service, the fare could be increased by a factor of 3.5 while the productivity fell by a factor of only 3.1. However, on further improving the level of service, to that of a taxi, the fare could only be increased by a factor of 1.3 while the productivity fell by a factor of 2.1. This implies that the passengers would be willing to pay

TABLE 3

Comparison of the optimum operations

Service	Number of Vehicles	Vehicle size (seats)	Fare (pence)	Passengers carried per week	Productivity (passengers per vehicle hour)	Revenue (£/week)	Operating cost (£/week)	Net loss (£/week)	User benefits (£/week)	Resource cost (£/week)	Net social benefit (£/week)
Profit maximising services											
Taxi	1	4	18	220	4.5	42	128	86	48	125	- 77
Shared taxi	1	4	14	470	9.6	71	137	66	85	133	- 48
Dial-a-bus	1	12	14	470	9.6	71	154	83	85	164	- 79
Original minibus	1	12	4	1110	22.6	47	95	48	77	106	- 28
Modified minibus	1	12	4	1450	29.6	67	95	28	106	106	0
Benefit maximising services											
Taxi	1	4	18	220	4.5	42	128	86	48	125	–77
Shared taxi	1	4	12	540	11.0	69	137	68	86	133	–47
Dial-a-bus	1	12	12	540	11.0	69	154	85	86	164	–78
Original minibus	1	16	3	1550	31.7	49	100	51	96	113	–17
Modified minibus	1	18	3	2070	42.2	72	102	31	132	117	+ 15

considerably more for the service offered by a dial-a-bus than they would for a fixed route service, but that they would be less willing to pay for further improvements to the service. This resulted in the dial-a-bus earning more revenue than any of the other services.

Turning now to the operating costs, it may be seen that the costs of operation for the demand responsive services were much higher than those for the fixed route services, this was because of the additional cost of control. The dial-a-bus was found to have appreciably lower costs when operated by a shared taxi than when a minibus was used, even though the minibus operation was credited with the bus grant and fuel rebate of a public transport system. The reason for this lies principally in the lower maintenance costs of the taxi. (See Appendix 5). However, the higher revenue earning capability of a shared taxi was not sufficient to recover its control costs and a fixed route minibus service was clearly found to be the least loss-making form of operation. That this result might be reversed for larger transport systems may be seen by considering a hypothetical situation of three adjacent towns, each of which requires identical transport to that of Carterton. On combining the figures in Table 3 and Appendix 5, the net loss on the three minibus services would then be £84 per week, while three shared taxis using a common control centre, would make a loss of only £74 per week. Such economies of scale would be reinforced in a real situation by the greater operational efficiency of a large shared taxi service.

When considering benefit maximising operations, it was found that the demand responsive systems were capable of generating almost four times more benefit per passenger than the fixed route services. This was brought about not only by the increased level of service that these systems provided, but also because the additional trips generated by the low fares on the fixed route services were of less benefit to the travellers than the trips that they were willing to make at a higher fare on the other services. However, these high level services are capable of carrying so few passengers that they were found to generate less total benefit than the fixed route services. Furthermore the resource costs were higher. The demand responsive systems therefore, showed far less net social benefit than the simple minibus services. In fact the modified minibus service was the only system which produced a positive net benefit.

It is of interest to note that the benefits associated with demand responsive services are only slightly greater for the benefit maximising services than for profit maximisation. This is because the passengers who are attracted to the service by the lower fares, impose a disbenefit of additional waiting time on the existing passengers.

7. SUMMARY AND CONCLUSIONS

A theoretical study has been made of the demand for public transport and of its supply in the town of Carterton, Oxfordshire. Three alternative transport systems were considered involving five distinct modes of operation: two different minibus routes, a dial-a-bus service, a shared taxi service and a conventional taxi service.

A demand model was developed and was shown to be in close agreement with observations for three different levels of public transport in the town. This model was used, together with supply models, to determine the optimum fare and level of service for each mode of operation. This optimisation was carried out for both commercial (profit maximising) and social (net benefit maximising) objectives.

The most important observation made in this study is that, because of the small size of the town, any internal transport service is in direct competition with walking and therefore has very few captive passengers. This observation underlies nearly all of the specific findings in this report which are listed below.

1. The demand for any internal service is shown to be very sensitive to the fare charged and to the level of service provided.

2. Because of the low fares necessary to compete with walking, none of the transport systems considered could generate sufficient revenue to cover their operating costs.

3. The dial-a-bus service was shown to be capable of earning more revenue than any other operation. However, the high cost of control made it less profitable (more loss making) than a fixed route service. A simple analysis shows that this situation might be reversed for a larger transport system which could share the control overheads between three or more vehicles.

4. It has been shown that a well designed fixed route service could cover some 70 per cent of its operating costs out of revenue and would require a subsidy of about £1400 per annum (in July 1974 prices).

5. A general result found was that the social objective would be best met by operating the services at lower fares and higher capacities than were required to minimise their financial loss.

6. It was found that only a well designed fixed route service could operate at a positive net social benefit (of about £750 per annum). This service showed a 12 per cent social rate of return and its operation might therefore be justified on social grounds, although it would require an operating subsidy of some £1600 per annum.

7. The only social benefits included in this study were the benefits accruing to the users of the services. These benefits are therefore mainly savings in time on journeys which would previously have been made on foot. Such savings have been assessed in accordance with accepted procedures, but it may be questioned as to whether society should invest resources in diverting relatively short trips from walk on to public transport.

8. The findings in this Report show that internal public transport services may be more viable and beneficial in larger towns where the trips are of longer distance and the competition from walk correspondingly less severe.

8. ACKNOWLEDGEMENTS

The work described in this report was carried out in the Public Transport Division (Head of Division: Mr M Grimmer) of the Transport Operations Department of TRRL.

9. REFERENCES

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10. APPENDIX 1

OPTIMISATION OF THE ORIGINAL FIXED ROUTE MINIBUS SERVICE

10.1 Survey results[1]

During the week of the survey 667 trips were made on the service. 133 of them were made by infants under the age of 5 and were not included in the calibration of the demand model since they involved no independent travel decision, but they influence the analysis in that approximately half of them were made by infants over the age of three years who paid a half fare. For the optimisation of all of the alternative transport systems it was assumed that these trips comprise a constant proportion of the total. Similarly, it was assumed that the proportion of trips made by children in the age group 3–14 years remained constant. Taken together these trips increased the fare yield to 1.056 times the adult fare.

Of the remaining 534 trips, 38 would not have been made if the service had not been provided, and 496 were diverted from other modes (including walking). The modal split model is concerned only with the diverted trips and it was assumed that, as the service was modified, the proportion of generated trips remained constant at 7.6 per cent of the diverted trips. It is thought probable that, with improvements to the service, the proportion of generated trips would increase. This assumption is therefore likely to provide an underestimate of the amount of generation, at lower fares. However, in the absence of further information, it is the least objectionable assumption and results in simplification of the model since the consumer surplus of the generated trips is simply 7.6 per cent of that of the diverted trips.

10.2 Capacity limitation

The service comprised four 15 minute tours per hour operated in a clover-leaf pattern. A total of 198 tours were operated each week. During the survey week the maximum number of passengers carried on a single tour was 18. Ninety per cent of the passengers using the service made trips to or from the crossroads at the town centre and, for the tour of highest demand, four of the passengers were taken from the crossroads to their homes and the remaining 14 were carried to the town centre. With the flow so heavily biased in one direction, there is only a slight chance that the vehicle contained more than 14 passengers at any point on the tour. It is therefore concluded that a 14-seat vehicle would have been sufficient to cope with the demand. Each of the 198 tours was divided in this way into two "half tours" (inbound to and outbound from the town centre) and a histogram of the number of seats required on each half tour is shown in Figure 1 of this Appendix.

In order to optimise the vehicle size it was necessary to determine the proportion of the demand which could be carried by a vehicle with insufficient capacity to satisfy the total demand. For the observed service this was obtained directly from Figure 1 by taking the vehicle capacity as an upper limit on the number of trips that could be carried on any half tour. This resulted in the curve of Figure 2 which shows the proportion of the demand which could be carried by vehicles of different size. (The vehicle size is defined by the number of seats and in Figure 2 it is shown as the proportion of the weekly demand which could be accommodated on the vehicle).

A further assumption is necessary in order to generalise this procedure to a situation in which the total demand has been changed by, say, a fare increase. It is assumed that the change in total demand results from a proportional change in the demand for all of the half tours, so that the shape of the demand histogram of Figure 1 remains unchanged. Making this assumption, Figure 2 may be used to determine the proportion of any total demand that will be satisfied. This figure shows that a vehicle of capacity one fortieth of the weekly demand would be capable of carrying almost all of the desired trips. However, there would be very few journeys for which this capacity was required and a vehicle of half of this size would fail to satisfy only 6.5 per cent of the demand.

10.3 Fare optimisation

The demand for the service was calculated at a range of fares and the results are presented in Table 1.

APPENDIX 1 TABLE 1

The variation of demand with fare

Fare (pence)	Demand (trips per week)					
	Diverted				Generated	Total
	Bus	Walk	Car	Total		
0	67	3614	903	4584	348	4932
1	56	2631	595	3282	249	3531
3	33	1347	267	1647	125	1772
6	17	457	88	562	43	605
9	6	161	22	189	14	203
20	0	6	0	6	0	6
25	0	0	0	0	0	0

The most striking aspect of these results is the very high fare elasticity*. The elasticity at the various fares was obtained graphically and is shown in Figure 3, from which it may be seen that the elasticity at a fare of 6p has the value - 2.1. This is considerably greater than is normally found for public transport services and implies that the number of people using the service would be very sensitive to the fare charged. This is because the minibus serves only short journeys which many people would choose to make on foot rather than pay higher bus fares. (This is expressed quantitatively by the cross-elasticity of the minibus service with respect to walking, which at a fare of 6p comprises 75 per cent of the total fare elasticity). The magnitude of this fare elasticity is confirmed experimentally by the demand for the modified minibus service which was derived from the original one by a number of operational improvements. At the same time the fare was increased from 6p to 10p. The net effect has been to halve the patronage and may well have resulted in a loss of revenue.

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* The fare elasticity of mode x at fare f_x is defined by $e_x = \frac{f_x}{n_x} \frac{\partial n_x}{\partial f_x}$, where n_x is the demand at fare f_x and $\frac{\partial n_x}{\partial f_x}$ is the gradient of the demand curve at this fare. In the limit of small changes, a fare increase of δ per cent increases the demand by e δ per cent. The cross elasticity of mode x with another mode y is defined by $e_{xy} = -\frac{f_x}{n_x} \frac{\partial n_y}{\partial f_x}$ so that $e_x = \sum_y e_{xy}$, where the summation runs over all other modes (including generation). e_{xy} therefore measures the proportion of the increased demand for mode x which is diverted from mode y by a reduction in the fare of mode x.

With such a high elasticity a reduction in the fare would have increased the demand sufficiently to more than off-set the loss in revenue from the passengers who were willing to travel at the higher fare. However, as shown in Figure 3, the magnitude of the elasticity falls as the fare is decreased and has the value -1, corresponding to maximum revenue, at a fare of 3p. This would also be the profit maximising fare if the cost of provision of the service were independent of the number of trips carried. The joint optimisation of fare and vehicle size is considered in Section 10.5 of this appendix.

However, the choice of vehicle size can only be made when a service is initially designed and this choice, together with its concomitant investment, will have been made for a service that is already being provided. Thus, in this case, it is of interest to know what fare should have been charged in order to minimise the loss made by operating the original minibus service with a 15 seat vehicle. The details of the fare optimisation for this case are shown in Table 2.

APPENDIX 1 TABLE 2

Fare optimisation for the operation of a 15 seat vehicle

Fare (pence)	Demand Trips/week	Vehicle size (% of weekly demand)	Trips carried		Revenue (£ per week)	Net loss (£ per week)
			% of demand	No. per week		
0	4932	0.30	54.4	2683	0	99
1	3531	0.42	65.5	2313	24.4	75
2	2470	0.61	77.7	1919	40.5	58
3	1772	0.85	86.2	1527	48.3	51
4	1245	1.20	92.8	1155	48.7	50
5	874	1.72	97.2	850	44.8	54
6	605	2.48	99.9	604	38.2	61
7	419	3.58	100	419	30.9	68

The procedure for calculating the net loss was as follows:

1. A fare was selected
2. The demand at that fare was determined
3. The vehicle size was calculated as a percentage of the demand
4. The percentage of the demand that would be carried by that vehicle was read from Figure 2
5. The number of trips carried was multiplied by the fare and revenue yield to give the revenue
6. This revenue was subtracted from the cost of operating the vehicle.

The operating cost of £99 per week was taken from Appendix 5 and is the same as the cost of operating a 15 seat vehicle on the modified service*. It may be seen that the actual revenue (at a fare of 6p) was about 20 per cent below the maximum achievable, which would result from a fare of 4p.

10.4 The optimisation of vehicle size

The optimum vehicle size is that at which the marginal cost of providing additional capacity is equal to the marginal revenue that this would produce. Thus, if a smaller vehicle were operated the cost of providing an additional seat would be less than the additional revenue that it would produce; on the other hand if the vehicle were larger the additional cost would not be off-set by the additional revenue.

The operation of more than one vehicle was also considered, for both the financial and social objectives, but it was found that the additional revenue and social benefit that would be generated could not cover the costs of operating further vehicles.

10.5 Joint optimisation of fare and vehicle size

If the fare were reduced from 6p, both the demand and revenue would increase. However, in order to meet the increased demand a larger vehicle would be required and the operating costs would increase. The joint optimisation of fare and vehicle size is therefore equivalent to finding that combination of fare and vehicle size which maximises the profit. This was carried out numerically and it was found that the cost saving of operating a smaller vehicle dominated the reduction in revenue throughout the range of vehicle sizes considered and that the optimum sized vehicle would therefore be smaller than a 12 seat bus. This is because there were few tours which required the full capacity of a bus and that the additional revenue which a bus could generate on these tours did not cover the additional cost of operating a large vehicle. The characteristics of the least loss-making bus operation (a 12 seat vehicle at a fare of 4p) are shown in Table 3 of this Appendix.

10.6 Social objectives

All of the foregoing analysis relates to profit maximisation, and a similar analysis was necessary in order to determine the fare and vehicle size which would maximise the net social benefit. This differed from the optimisation with a financial objective only in the replacement of revenue by user benefits and of operators' costs by resource costs. As described in Section 4.3, the consumer surplus, required for the user benefits, measures the total amount that the passengers would be willing to pay for the particular service being provided. In the present case, this is the amount that they would be willing to pay for a service with a particular level of capacity constraint.

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* This cost is considerably lower than the published cost of operating the original minibus service, which included the cost of redundant control facilities. The lower cost was used in order to permit direct comparison between the two minibus services.

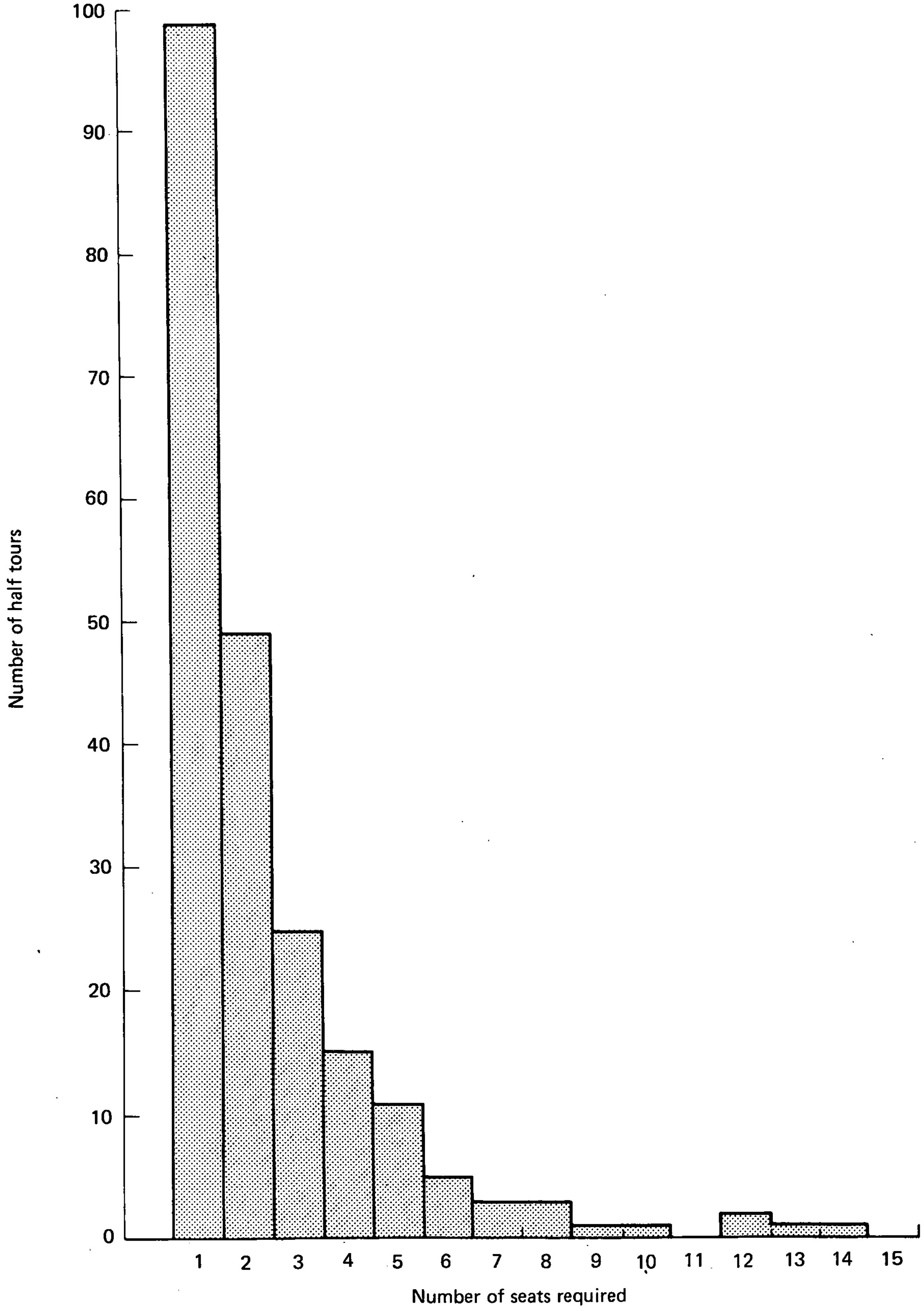
On carrying out this analysis it was found that the original minibus service at a fare of 6p was operating at a net social loss of £57 per week. This loss would have been minimised by operating the 15-seat vehicle at a fare of 2p. At this fare, about 1900 passengers would have been carried each week at a net cost of £58 per week and a net social loss of £10 per week. The joint optimisation of fare and vehicle size indicate that the net social loss would be minimised by operating a 26-seat vehicle at a fare of 1p. This service would have carried about 2900 passengers at a net social loss of £6 per week. It should be remembered that, in making this calculation, it was assumed that a larger vehicle could operate over the minibus route and that it could complete four tours in an hour. These assumptions are probably not valid for such a large vehicle stopping to pick up so many passengers. This calculation should therefore be taken merely to indicate that the social cost of the service would be reduced by operating a larger vehicle at a lower fare than that which minimises the financial loss. Taking these factors into account, it is estimated that, if the objective had been to maximise social benefit, the most practicable operation would have employed a 16-seat vehicle at a fare of 3p. The characteristics of this service are compared with those of the least loss-making operation in Table 3 of this Appendix.

It should be noted that both the financial and social losses made by operating this service are far more sensitive to the fare than they are to the size of the vehicle. Thus, from the operational viewpoint, it is essential to select the correct fare to meet the desired objective, but it is only necessary to ensure that the vehicle has sufficient capacity.

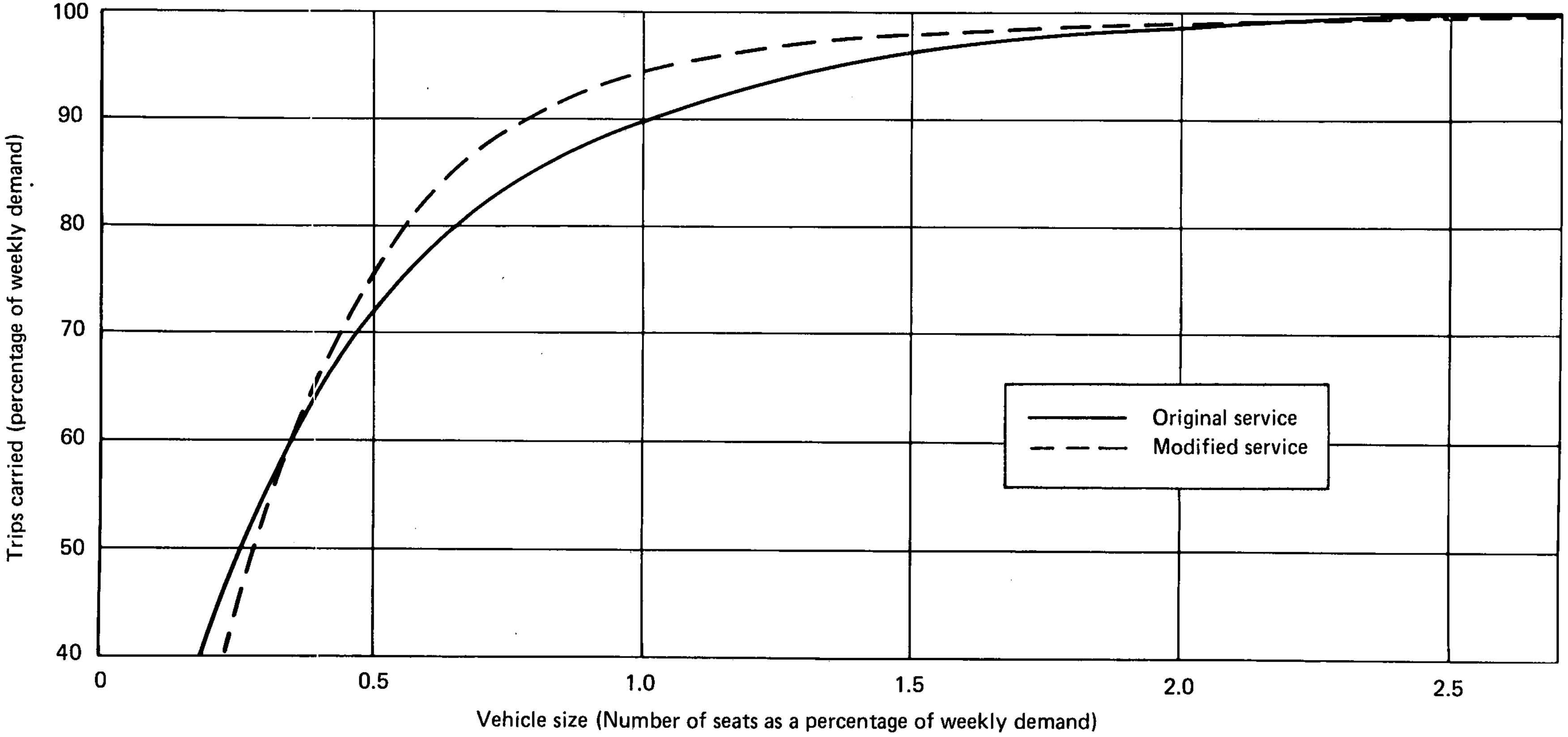
APPENDIX 1 TABLE 3

Characteristics of the original minibus services with the fare and vehicle size optimised

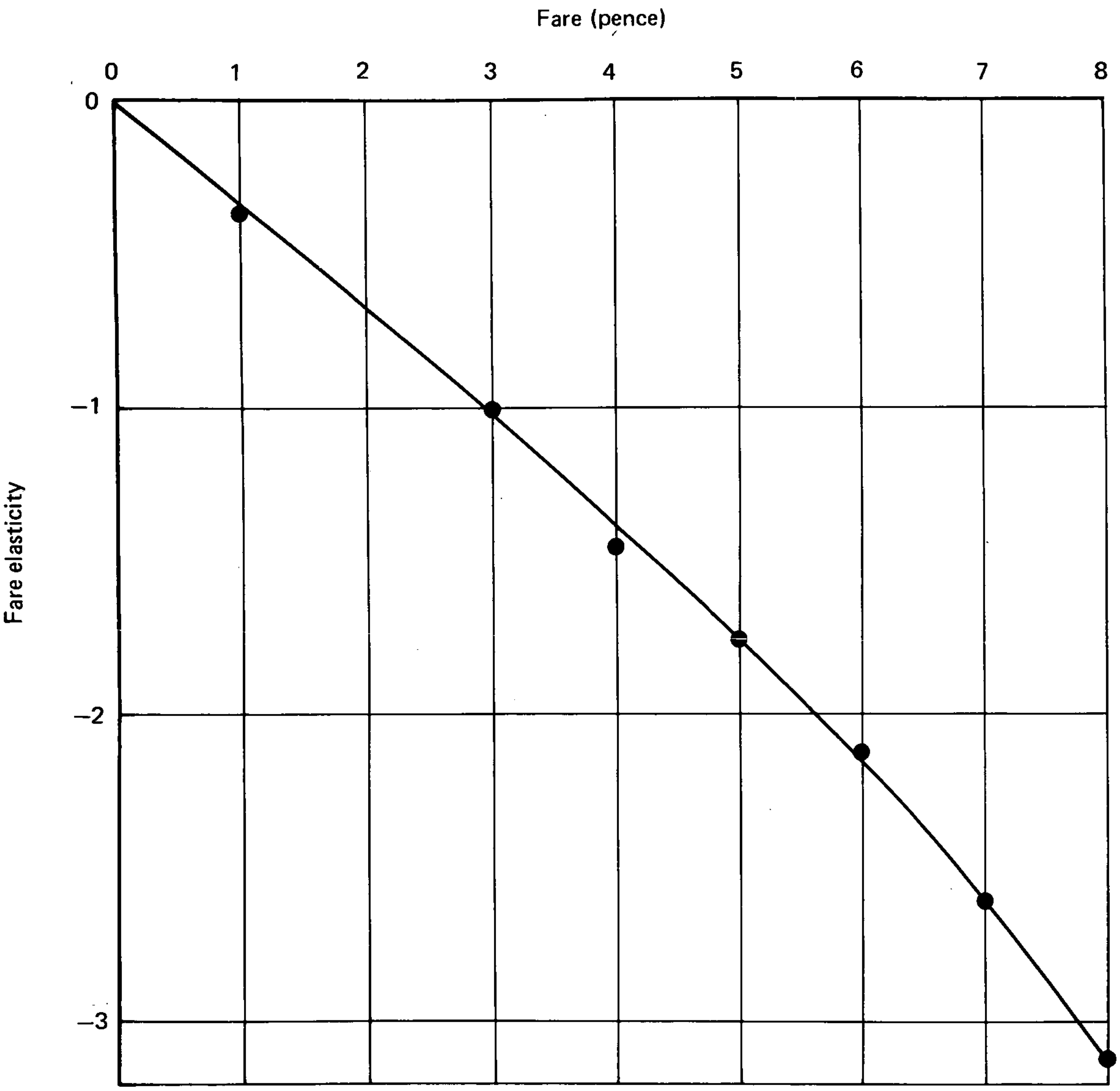
Characteristic	Loss minimising service	Benefit maximising service
Fare (pence)	4	3
Vehicle size (seats)	12	16
Productivity (passengers/vehicle hour)	22.6	31.7
Revenue (£/vehicle hour)	0.96	1.00
User benefits (£/vehicle hour)	1.58	1.96
Operating costs (£/vehicle hour)	1.94	2.04
Resource costs (£/vehicle hour)	2.16	2.31
Net loss (£/vehicle hour)	0.98	1.04
Net social loss (£/vehicle hour)	0.58	0.35



Appendix 1 Fig. 1 OBSERVED DISTRIBUTION OF THE DEMAND FOR SEATS ON THE ORIGINAL MINIBUS SERVICE



Appendix 1 Fig. 2 PROPORTION OF SATISFIED DEMAND FOR FIXED ROUTE MINIBUS SERVICES



Appendix 1 Fig. 3 FARE ELASTICITY FOR THE ORIGINAL MINIBUS SERVICE

11. APPENDIX 2

OPTIMISATION OF THE MODIFIED FIXED ROUTE MINIBUS SERVICE

11.1 Survey results

Ticket counts during April and May 1975 showed that the average number of tickets sold was 433 per week. On the basis of the composition of the ridership of the original minibus service, 11 per cent of these tickets were sold to children under the age of 5 years. The service therefore carried 385 passengers per week aged over 5 years. A count was also made of the number of passengers carried on each vehicle tour. A demand satisfaction curve was constructed from these numbers in exactly the same way as described in Appendix 1 for the original minibus service. The curves for both of the services are shown in Figure 2 of Appendix 1. On comparing these two curves it may be seen that, as the vehicle size is reduced, a larger proportion of the demand is carried on the modified service but that, once insufficient capacity is being supplied, its effects are more severe than for the original minibus service. These effects arise from the more even distribution of demand over the vehicle tours of the modified service, and indicate that it is more efficient than the original one.

11.2 Fare optimisation

As described above, in April 1975 the service was carrying 385 passengers per week at a basic fare of 10p. In the theoretical models all costs were measured in June 1974 prices and, in comparing the model predictions with the observed patronage, it was necessary to make some allowance for inflation during the period June 1974 to April 1975. The index of retail prices[7] shows that transport costs increased from 110.9 in June 1974 to 138.1 in April 1975 (January 1974 = 100). Inflation during this period was therefore 25 per cent and it is therefore assumed that a 10p fare in April 1975 is equivalent to an 8p fare in April 1974. The model predicted a patronage of 395 passengers per week at this fare, in excellent agreement with the observations. At this fare the service was operating at a net loss of £63 per week and at a net social loss of £66 per week.*

On optimising the fare for a 15 seat vehicle it was found that the revenue would have been maximised at a fare of 4p. At this fare the service would have carried about 1600 passengers each week at a net loss of £29 per week, and would have shown no net social loss. This calculation, therefore, shows that by halving the fare it would have been possible to halve the subsidy necessary to operate the service and to generate sufficient social benefit to cover the resource costs.

11.3 Joint optimisation of fare and vehicle size

The joint optimisation of fare and vehicle size was carried out in exactly the same way as for the original minibus service, and produced similar results. It was found that the least loss-making bus operation would employ the smallest possible bus (12 seats) operating at a fare of 4p. The most socially beneficial service would require a 35 seat vehicle operating at a fare of 1p. This service would generate a positive net social benefit of £33 per week which is equivalent to a 23 per cent social rate of return. However, as discussed in Appendix 1, for the original minibus service, a vehicle of this size could not be operated over the specified route and, with the objective of maximising social benefit, it is estimated that the most practicable operation

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* Unless otherwise stated, all sums are quoted in June 1974 prices.

would employ an 18 seat vehicle at a fare of 3p. This service would carry some 1000 passengers per week at a net social benefit of £15 per week and would require a subsidy of £31 per week. The social rate of return would be 12 per cent. It is therefore possible to justify, in social terms, the operation of a subsidised minibus service providing transport solely within the town. The characteristics of this service are compared with those of the loss minimising operation in Table 1 of this Appendix.

APPENDIX 2 TABLE 1

Characteristics of the optimum modified minibus services

Characteristic	Loss minimising service	Benefit maximising service
Fare (pence)	4	3
Vehicle size (seats)	12	18
Productivity (passenger/vehicle hour)	29.6	42.2
Revenue (£/vehicle hour)	1.37	1.47
User benefits (£/vehicle hour)	2.16	2.68
Operating costs (£/vehicle hour)	1.94	2.09
Resource costs (£/vehicle hour)	2.16	2.38
Net loss (£/vehicle hour)	0.57	0.62
Net social benefit (£/vehicle hour)	0	0.31

12. APPENDIX 3

OPTIMISATION OF A MANY-TO-MANY DIAL-A-BUS SYSTEM

12.1 The equilibrium between supply and demand

The operation of this type of service is described in Section 3.3 and the general form of the supply model is discussed in Section 4.2.2. In applying this supply model to the case of Carterton, the town was approximated by a square of area 1.4 square miles and the mean car journey time for the average dial-a-bus trip was found to be 3.4 minutes. With these values the sum of waiting and riding time, t, is related to the demand, N, in trips per week, and the fleet size, n, by the expression

$$t = 3.4\left\{1 + \left[\frac{(1.15 + 0.00174N)}{n}\right]^2\right\} \qquad (1)$$

The demand was obtained by inserting the characteristics of a dial-a-bus service into the demand model. Predictions were therefore made of the ridership of a mode providing a doorstep service (no access or egress time) at a range of fares, (from 5p to 20p), and with wait times (from 5 minutes to 20 minutes) and an inter-zone ride time that was a multiple of the corresponding ride time by car (from twice to four times). The results of these calculations were shown to be well approximated by the expression*

$$N = N_0 e^{-\beta(f + 0.933w + 0.466r)} \qquad (2)$$

which relates the demand, N, in trips per week, to the fare, f, in pence, the mean waiting time, w, in minutes and the mean ride time, r, also in minutes. The values of the coefficients, N_0 and β were obtained by curve fitting and were 456820 per week and 0.262 per penny respectively.

It should be noted that the level of service in the supply equation is measured by t, the simple sum of the wait and ride times but that the demand is a function of a weighted sum of these times. This results from the behavioural basis of equation (2) according to which the travellers value waiting time twice as highly as time spent riding in a vehicle.

In order to obtain an equilibrium between supply and demand, equations (1) and (2) were solved simultaneously. These solutions were obtained graphically in exactly the same way as for the taxi service (Appendix 4) to obtain an equilibrium between the demand and waiting time for each fare and fleet size.

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* An equation of this form may be obtained by expanding the analytical form used in the demand model in powers of the independent variables. Such an expansion indicates that an equation of the form (2) should be valid for a minority mode.

12.2 The optimisation of fare and fleet size for the operation of 15-seat vehicles

The revenue from each of the equilibrium operations was calculated directly from the demand and revenue yield of 1.055 times the fare. This is shown as a function of fare and fleet size in Figure 1 of this Appendix, from which it may be seen that the revenue is maximised at a fare of 14p for the operation of one or two vehicles. The maximum revenue for the operation of a single vehicle was about £70 per week and for the operation of two vehicles was £117 per week. From the operating costs of a 15-seat vehicle, derived in Appendix 5, these correspond to net losses of £89 and £138 per week, respectively. Thus, for a service which offers a ride time of twice that of car, the least loss making operation employs one vehicle at a fare of 14p carrying approximately 470 passengers per week.

On integrating equation (2), the consumer surplus is given by

$$CS = \frac{1}{\beta}N \qquad (3)$$

and, if it is assumed that all trips are diverted from walk, the user benefits are given by

$$B = \left(f + \frac{1}{\beta}\right)N \qquad (4)$$

This equation gave values within 2 per cent of the accurately computed benefits from the demand model. These benefits are shown as a function of fare and fleet size in Figure 2 of this Appendix and the benefit maximising combinations are shown in Table 1 below.

APPENDIX 3 TABLE 1

The variation of maximum net benefit with fleet size
(for the operation of 15-seat vehicles)

Fleet	Benefit maximising fare (p)	Trips carried per week	Users benefits (£/week)	Resource costs (£/week)	Net social cost (£/week)
1	12	540	86	170	84
2	13	840	142	280	138
3	13	1110	186	390	204

With a ride time twice that of car, the most socially beneficial service uses a single minibus operating at a fare of 12p. Again, as seen in the optimisation of the fixed route services, the social objective requires operating at a lower fare and higher demand than that required by the financial objective.

The entire process of determining the optimum fare and fleet size was repeated for longer ride times. As noted earlier, the demand is less sensitive to ride time than to waiting time whilst the supply equation gives them equal weight. The equilibrium between supply and demand at longer ride times is therefore found at a higher demand. For the case of a single vehicle providing a ride time of three times that of car, the revenue maximising fare was found to be 15p. At this fare 490 passengers per week would be carried at an operating loss of £81. Similarly the social objective would be best achieved by operating at a fare of 13p, at which 560 passengers per week would be carried at a net social loss of £76. On comparing these figures with values corresponding to a shorter ride time, it may be seen that the equilibrium between supply and demand is relatively insensitive to the multiple of car ride time that was used. The figures quoted in Section 5 are based on a ride time of twice that of car.

12.3 The optimisation of vehicle size

As explained in Section 4.2.2, it was not possible to optimise accurately the size of vehicle which should be operated on a dial-a-bus service. This optimisation was therefore restricted to determining whether the demand would necessitate the use of minibus or whether it could be served by operating shared taxis. As shown in the previous section, the optimum operation of a single vehicle requires carrying about 500 passengers per week. In order to determine whether this demand could be served by a taxi, it is necessary to determine the probability that the vehicle would be required to contain more than four passengers. Now, the average arrival rate is one passenger every 6 minutes and the average ride time is about 7 minutes. Since the distribution of the demand with time is approximately Poisson, it may be shown that there is a probability of less than 1 per cent that the vehicle would be required to carry 5 or more passengers at one time. This result would be modified by including distributions of group size and of passenger riding times but it may be reasonably concluded that a taxi is capable of coping with nearly all of the demand.

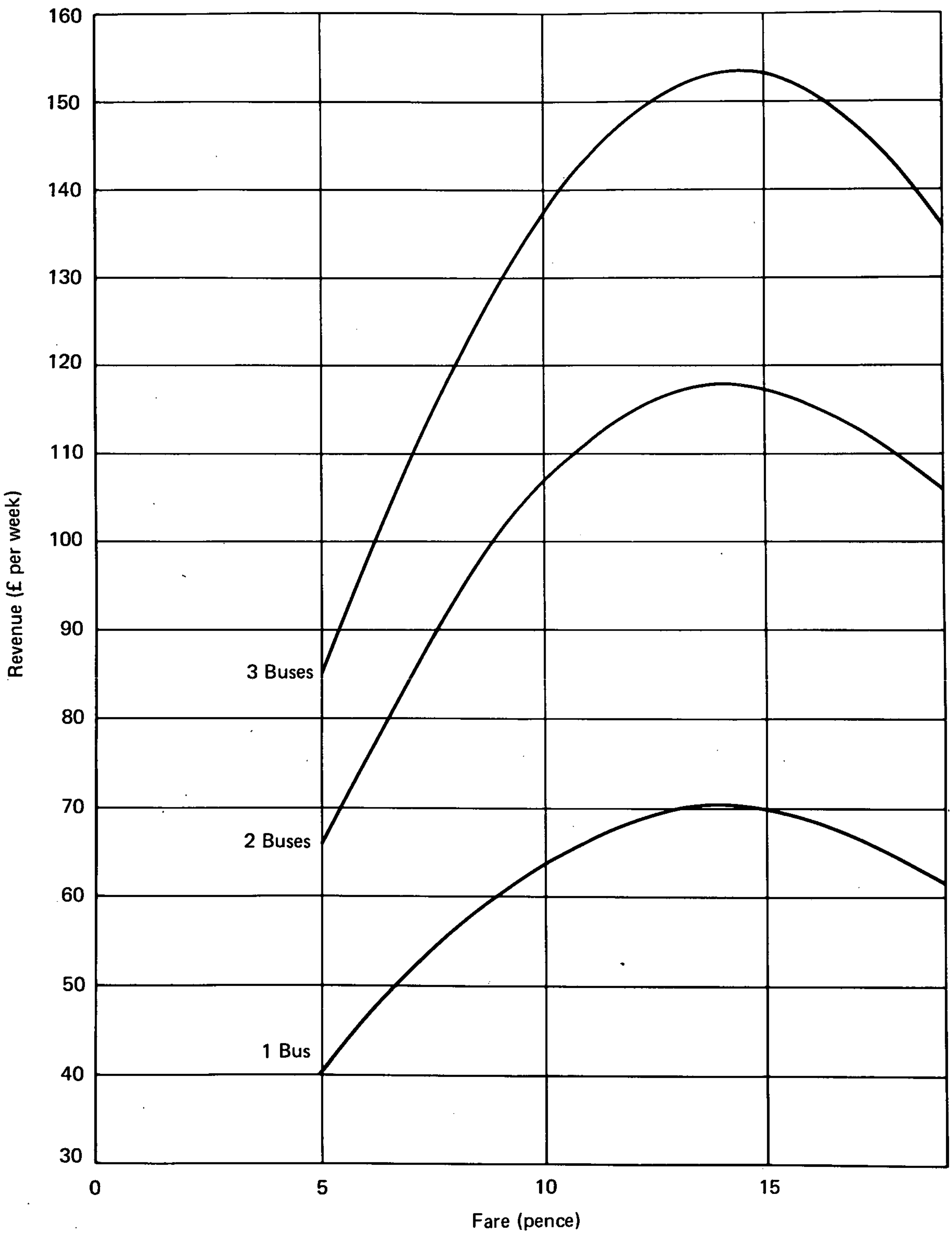
The financial and social assessments were therefore repeated for the operation of shared taxis and it was again found that a single vehicle would best meet both objectives.

Table 2 summarises the performance of the optimum dial-a-bus systems when operated using a taxi or minibus. In order to facilitate a comparison with the other transport systems the quoted figures were calculated on a per vehicle hour basis.

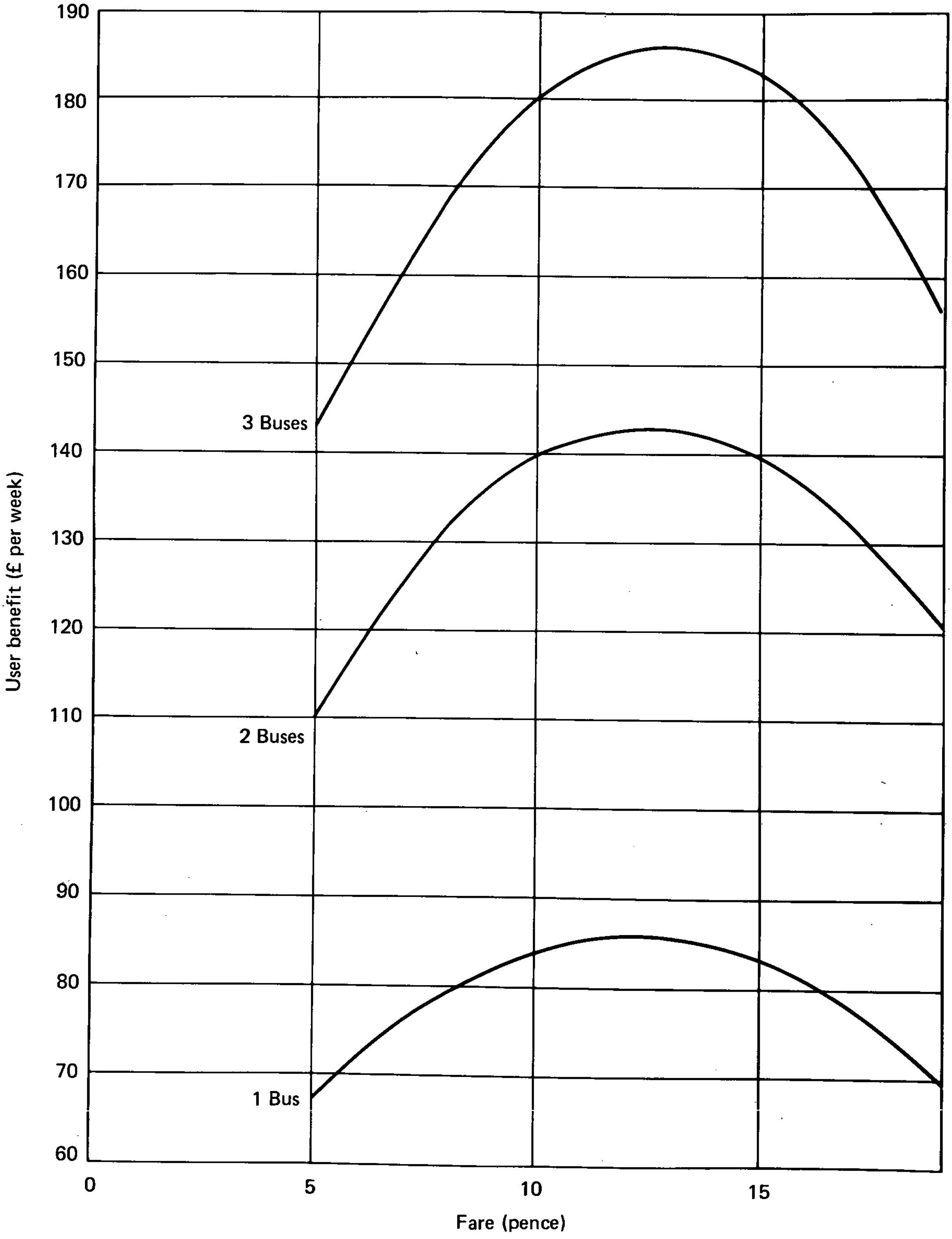
APPENDIX 3 TABLE 2

Characteristics of the optimum dial-a-bus systems

Characteristic	Minibus		Taxi	
	Loss Minimising Service	Benefit Maximising Service	Loss Minimising Service	Benefit Maximising Service
Number of vehicles	1	1	1	1
Vehicle size (seats)	12	12	4	4
Fare (pence)	14	12	14	12
Productivity (passenger/vehicle hour)	9.6	11.0	9.6	11.0
Revenue (£/vehicle hour)	1.44	1.40	1.44	1.40
User benefits (£/vehicle hour)	1.73	1.75	1.73	1.75
Operating cost (£/vehicle hour)	3.14	3.14	2.79	2.79
Resource cost (£/vehicle hour)	3.35	3.35	2.71	2.71
Net loss (£/vehicle hour)	1.70	1.74	1.35	1.39
Net social loss (£/vehicle hour)	1.62	1.60	0.98	0.96



Appendix 3 Fig. 1 VARIATION OF REVENUE WITH FARE FOR DIAL-A-BUS OPERATION
(Twice car ride time)



Appendix 3 Fig. 2 VARIATION OF BENEFIT WITH FARE FOR DIAL-A-BUS OPERATION
(Twice car ride time)

13. APPENDIX 4

OPTIMISATION OF A TAXI SERVICE

The optimisation of a taxi service requires the determination of the fare and fleet size which maximises the objective function. This involves finding an equilibrium between the supply of the service and the demand for it, and is a similar problem to that of optimising a dial-a-bus service, discussed in Appendix 3, for which the supply, as measured by the passenger waiting time, varies with the demand, whilst the demand is itself a function of the waiting time. It was therefore necessary to set up equations for the supply and demand, and to simultaneously solve these equations for the equilibrium waiting time and demand, for each fleet size and fare. These equilibrium solutions were then searched for the profit or social benefit maximum.

The demand was obtained by inserting the characteristics of a taxi service into the demand model, and predicting the demand for a range of fares and waiting times. The service was specified as providing a ride time equal to that of a car with no access or egress time. The resulting demand was shown to be closely approximated by the form

$$N = N_0 e^{-\beta(f + 0.933w)} \qquad (1)$$

where f denotes the fare in pence and w the passenger waiting time in minutes. The coefficients N_0 and β were found to have the values 355358 per week and 0.272 per pence, respectively*.

As discussed in Section 4.2.3 the mean passenger waiting time was obtained by the application of queueing theory. For a service operated by one taxi, the queueing time was obtained from the Pollaczek-Khintchine formula[6]

$$Q = \frac{1}{2}\left(\frac{\rho}{1-\rho}\right)(1 + v^2)\ t \qquad (2)$$

where t is the mean service time and v is the coefficient of variation (standard deviation divided by the mean) of the service times. ρ is the traffic intensity and is related to the average arrival rate, λ, (number of arrivals in unit time), by

$$\rho = \lambda t \qquad (3)$$

Equation (2) holds for any distribution of service times but is exact only for a Poisson distribution of arrivals. In the present case, the distribution of demand is approximately Poisson and equation (2) provides a good approximation to the queuing time.

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* It may be noted that this value of the coefficient β is larger than that used for a dial-a-bus service. This results from curve fitting at the higher fares that are appropriate to a taxi service and from the difference in mean journey length.

The mean service time is made up of three components: the average time taken to reach the pick-up address, the time taken to carry the passenger from his origin to his destination and the dead time spent receiving instructions, waiting for the passenger and collecting the fare. With a mean journey length of 1.1 miles and an average road speed of 15 mph, the average passenger journey time is 4.4 minutes. Assuming that the revenue mileage of a taxi is equal to 62 per cent of the total mileage[8], each passenger journey involves the operation of 0.67 empty miles. Therefore, on average, 2.7 minutes are spent reaching the pick-up address. Allowing a further 2 minutes of dead time per passenger journey the mean service time is equal to about 9 minutes.

The coefficient of variation of the service times was calculated for the idealised case in which the trip ends are uniformly distributed in a square area. This calculation gave the value 0.47. The coefficient of variation of the passenger ride times was calculated, from the demand model, to be 0.54. The coefficient of variation of the service times was therefore taken to be 0.50.

On inserting these values into equation (2), the passenger waiting time is given by the expression

$$W = \left(\frac{0.014N}{1 - 0.0026N} \right) + 2.7 \qquad (4)$$

where N is the demand in trips per week and the waiting time, W, is measured in minutes*.

The queueing theory problem cannot be solved exactly for the operation of more than one taxi. However, an approximate solution may be obtained[6]. For the operation of n taxis this takes the form:

$$Q^{(n)} = \frac{1}{2} E^{(n)} (1 + v^2)\, t \qquad (5a)$$

where $E^{(n)}$ is given by

$$E^{(n)} = \frac{p\rho^n}{n.n!\,(1 - \rho/n)^2} \qquad (5b)$$

with

$$\frac{1}{p} = \frac{\rho^n}{n!} \; \frac{1}{(1 - \rho/n)} + \sum_{r=0}^{n-1} \frac{\rho^r}{r!} \qquad (5c)$$

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* In deriving equation (4) a group size of 1.2 was assumed when relating λ, which measures the number of passenger carrying journeys, to the demand, which measures the number of individual trips made on the service.

On applying these equations to the case of two taxis, the passenger waiting time is given by the expression

$$W^{(2)} = 2.7 + 5.63 \frac{\rho^2}{(4 - \rho^2)} \qquad (6)$$

with $\rho = 0.0026N$

Having obtained these supply equations they were solved simultaneously with the demand equation to obtain an equilibrium between the demand and waiting time for each fare and fleet size. These solutions were obtained graphically as the intersections of the two sets of curves. Thus, in Figure 1 of this Appendix the supply curves show the variation of waiting time with demand and the demand curves show the variation of demand with waiting time. As an example of the use of these curves it may be seen that, for the operation of two vehicles at a fare of 15p per passenger, 580 trips per week would be carried with a mean waiting time of 9.2 minutes. The profit and net social benefit were then calculated for operation at each of these points of equilibrium and the most profitable and the benefit maximising solutions were found. Both the commercial and social objectives were best met by the operation of a single vehicle at a fare of 18p*. The characteristics of the optimum service are shown in Table 1.

APPENDIX 4 TABLE 1

Characteristics of the optimum taxi service

Characteristic	Profit and net benefit maximising service
Fare (pence)	18
Productivity (passengers/vehicle hour	4.5
Revenue (£/vehicle hour)	0.85
User benefits (£/vehicle hour)	0.97
Operating costs (£/vehicle hour)	2.61
Resource costs (£/vehicle hour)	2.55
Net loss (£/vehicle hour)	1.76
Net social loss (£/vehicle hour)	1.57

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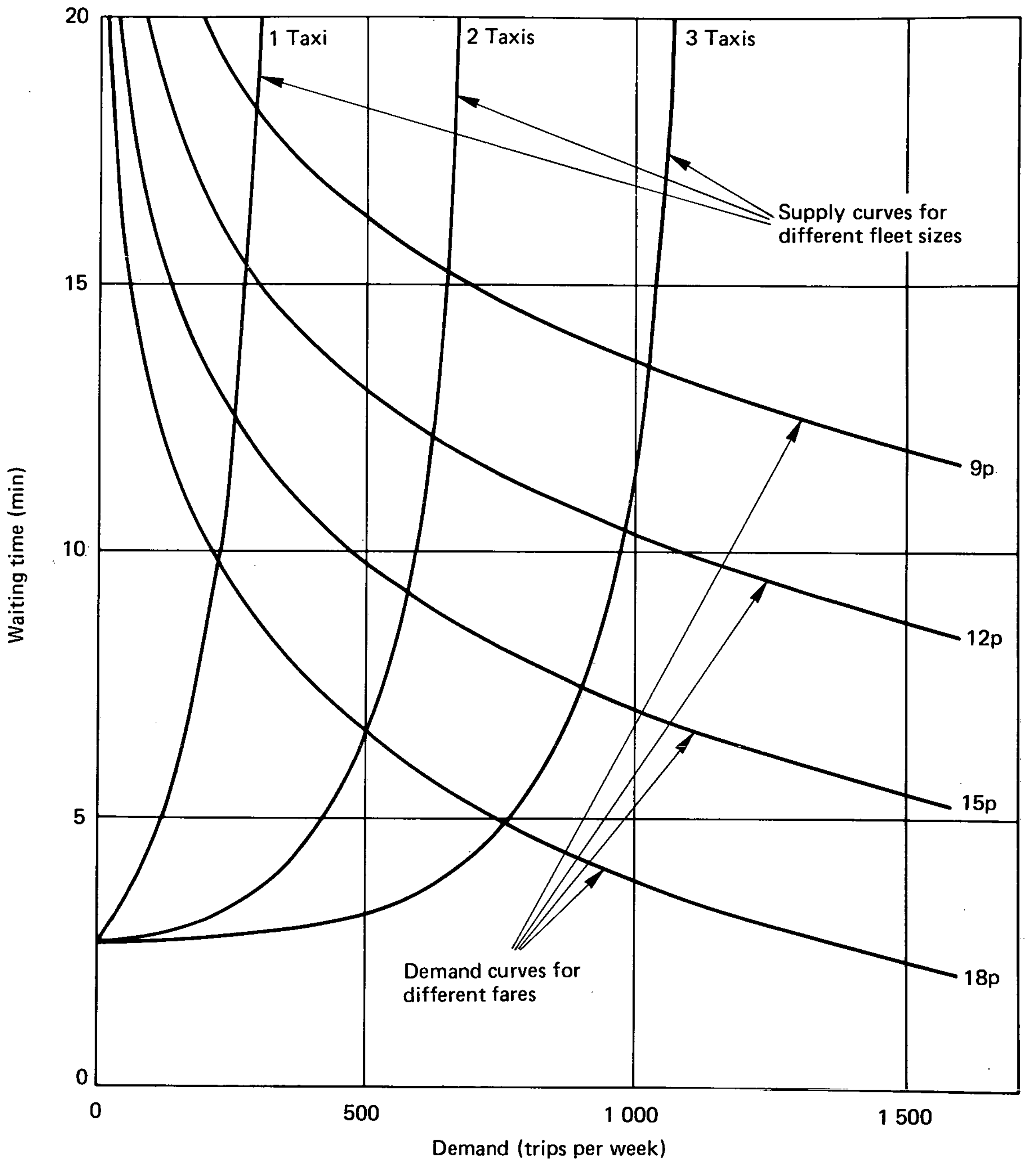
* In order to present results which are directly comparable with those for the other transport systems, the fare per passenger is quoted. Allowing for a group of 1.2 this is equivalent to a fare of 22p per journey.



Appendix 4 Fig. 1 SUPPLY AND DEMAND CURVES FOR TAXI OPERATION

14. APPENDIX 5

OPERATING COSTS

14.1 General considerations

Vehicle operating costs were required for two purposes. Firstly, they were necessary for the evaluation of the saving in resources that would result from a saving in car mileage. Thus, for a trip attracted from car on to public transport there is a benefit, in addition to the time saving of the traveller, equivalent to the resource cost of making the trip by car. For the systems considered in this report, these resource savings never exceed three per cent of the benefits from time saving, and they therefore had only a marginal effect on the analysis.

More importantly, operating costs were also required in order to determine the cost of provision of each of the services. For an analysis carried out on the basis of profit maximisation the costs incurred by the operator were required and for a social assessment the resource costs were needed. In the first case there was the additional complication of whether the service was to be run by a public transport undertaking or by a private operator. A public transport operator receives a 50 per cent grant on the purchase of new vehicles and a fuel rebate of 22.5p/gallon, which are not paid to a private operator.

The way in which the components of the operating cost have been estimated for each of these cases is shown in Table 1 of this Appendix, and the costs for the operation of each type of system are considered in the following sections. All figures quoted are in July 1974 prices and, unless otherwise stated, are based on the tables of Reference 9. The actual costs used in this report for the operating costs of services in Carterton were obtained from these figures with the substitution of local costs where they were known.

APPENDIX 5 TABLE 1

THE COMPONENTS OF OPERATING COSTS

Vehicle independent standing costs

	Operator's Costs	Resource Costs
Control centre, base radio and other equipment	Construction and purchase costs amortised at 12 per cent over 10 years	As Operator's costs less tax amortised at 10 per cent
Telephone	Rental	Rental less tax
Base radio maintenance	Contract cost (7.5 per cent of capital cost per annum)	As operator's costs less tax (assumed 100 per cent labour)
Controllers wages	Local costs if known	As operator's costs

TABLE 1 (Continued)

Vehicle standing costs

	Operator's Costs	Resource Costs
Licences	According to vehicle size	Nil
Rent and rates for garage	Local costs if known	As operator's costs (rates taken as a proxy for resource costs of local government services)
Insurance	According to vehicle size	As operator's costs less tax (taken as a proxy for accident costs)
Interest and depreciation* of vehicles	Purchase price less tyres and 10 per cent residual value amortised at 12 per cent over vehicle life	Purchase price less tyres, 10 per cent residual value and tax amortised at 10 per cent over vehicle life
Vehicle radio*	Purchase price amortised at 12 per cent over 8 years	Purchase price less tax amortised at 10 per cent over 8 years
Vehicle radio maintenance	Contract cost (7.5 per cent of capital cost per annum)	Contract cost less tax (assumed 100 per cent labour cost)
Drivers' wages	Local costs if known	As operators' costs

Vehicle running costs

	Operator's Costs	Resource Costs
Fuel†	Retail cost	Retail cost less tax and excise duty
Lubricants and tyres	Retail cost	Retail cost less tax
Maintenance	According to vehicle size	As operator's costs (assumed 100 per cent labour)
Track costs	Nil	According to size

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* A public transport undertaking receives a 50 per cent grant on the purchase price of these items.

† A public transport undertaking receives a rebate of the excise duty on fuel (22.5 pence per gallon).

14.2 The costs of bus operation

The components of the costs of bus operation are shown in Tables 2 and 3 of this Appendix. The figures quoted in Table 2 take account of capital grants and the fuel rebate and are therefore appropriate for use by a public transport operator.

APPENDIX 5 TABLE 2

The costs of the bus operation by a public transport operator

Number of Seats	12	15	31	45	57	76	80
VEHICLE STANDING COSTS (£ per week)							
Licences	0.26	0.28	0.39	0.54	0.67	0.77	1.00
Rent and rates (garage)	3.01	3.07	3.37	3.37	3.49	3.60	3.60
Insurance	3.33	3.52	4.56	5.26	6.39	7.68	8.23
Interest and depreciation	6.62	9.07	10.12	12.54	15.54	19.08	19.97
Driver's wages	44.64	44.64	44.64	44.64	44.64	44.64	44.64
Total:	57.86	60.58	63.08	66.35	70.73	75.77	77.44
RUNNING COSTS (pence per vehicle mile)							
Fuel	1.91	2.03	2.18	3.05	3.39	3.81	3.81
Lubricants	0.13	0.14	0.16	0.17	0.18	0.19	0.19
Tyres	0.24	0.24	0.72	1.00	1.00	1.00	1.00
Maintenance	3.11	3.37	4.69	5.62	5.67	6.22	6.37
Total:	5.39	5.78	7.75	9.84	10.24	11.22	11.37
SUPPORTING DATA							
Cost of fuel (pence per gallon)	30.5	30.5	30.5	30.5	30.5	30.5	30.5
Fuel consumption (m.p.g.)	16	15	14	10	9	8	8
Cost of tyres (£)	48	48	180	300	300	300	300
Mileage life of tyres	20,000	20,000	25,000	30,000	30,000	30,000	30,000
Life of vehicle (years)	3	5	6	6	8	10	10
Cost of vehicle (£)	2,115	4,300	5,590	7,003	10,337	14,317	14,967

APPENDIX 5 TABLE 3

The resource costs of bus operation

Number of Seats	12	15	31	45	57	76	80
VEHICLE STANDING COSTS (£ per week)							
Rent and rates (garage)	3.01	3.07	3.37	3.37	3.49	3.60	3.60
Insurance	3.08	3.26	4.22	4.87	5.92	7.11	7.62
Interest and depreciation	13.42	17.67	19.51	24.14	29.19	35.88	37.54
Driver's wages	44.64	44.64	44.64	44.64	44.64	44.64	44.64
Total:	64.15	68.64	71.74	77.02	83.24	91.23	93.40
RUNNING COSTS (pence per vehicle mile)							
Fuel	1.66	1.77	1.90	2.66	2.93	3.33	3.33
Lubricants	0.12	0.13	0.15	0.16	0.17	0.18	0.18
Types	0.22	0.22	0.67	0.93	0.93	0.93	0.93
Maintenance	3.11	3.37	4.69	5.62	5.67	6.22	6.37
Track costs	1.10	1.18	1.58	1.95	2.31	2.81	2.99
Total:	6.21	6.67	8.99	11.32	12.04	13.47	13.80
SUPPORTING DATA							
Cost of fuel (pence per gallon)	26.6	26.6	26.6	26.6	26.6	26.6	26.6
Fuel consumption (m.p.g.)	16	15	14	10	9	8	8
Cost of tyres (£)	44.4	44.4	166.7	277.8	277.8	277.8	277.8
Mileage life of tyres	20,000	20,000	25,000	30,000	30,000	30,000	30,000
Life of vehicle (years)	3	5	6	6	8	10	10
Cost of vehicle (£)	1,946	3,956	5,143	6,443	9,409	13,172	13,770

Figure 1 of this Appendix shows the variation with vehicle size of the cost of operating a single vehicle a distance of 670 miles per week (the observed mileage of the Carterton minibus services). This Figure shows a calculated operating cost of £99 per week for a 15 seat vehicle which is in excellent agreement with the observed cost of £103 per week[1]. For vehicles with up to about 40 seats, the variation of operating costs may be approximated by the linear equations:

$$C = 81 + 1.15s$$

$$R = 89 + 1.43s$$

where C and R are the costs to the operator and the resource costs, respectively, in £ per week, and s is the number of seats in the vehicle.

14.3 The costs of dial-a-bus operation

The costs of operating a dial-a-bus service differ in two respects from those of minibus operation. Firstly there is the cost of despatching the vehicles, comprising the costs of the control centre, the telephone, the base radio and the vehicle radios. Secondly, the distance travelled by each vehicle and therefore the vehicle running cost, varies with the demand. The components of the additional costs of control are shown in Table 4 below, for a system controlled by a single despatcher. The radio and telephone costs shown in this table are based on data from References 10 and 11.

APPENDIX 5 TABLE 4

Control costs for a dial-a-bus service

Component	Costs (£ per week)	
	Operator's Costs	Resource Costs
Control centre	12.65	11.63
Base radio	2.64	2.43
Base radio maintenance	1.11	1.11
Telephone rental	1.46	1.35
Controller's wages	44.0	44.0
Vehicle radio	1.47	1.35
Vehicle radio maintenance	0.55	0.55
Total vehicle independent standing costs	61.86	60.52
Total vehicle dependent standing costs	2.02	1.90

SUPPORTING DATA	£
Cost of control centre and equipment	3640
Cost of base radio	775
Telephone installation	76
Cost of vehicle radio	380

It was not possible, without simulation, to determine the variation of vehicle mileage with demand. However, the optimisation techniques used in this work ensure that the vehicles are used to maximum efficiency, and it may be assumed that the vehicles are never idle. The mean operating speed will therefore be the same as, or slightly higher than, that of the vehicles operating in the Harlow dial-a-bus service[12,13], namely 12 miles per hour. Each vehicle would therefore cover approximately 590 miles in a 49 hour operating week. With this assumption the costs of operating a dial-a-bus fleet of 15-seat vehicles are as shown in Table 5. This table also shows the costs of operating a fleet of shared taxis under the same assumptions but using costs appropriate to a private operator rather than those of a public transport undertaking.

APPENDIX 5 TABLE 5

The costs of dial-a-bus operation (£/week)

Fleet size	Minibus Operation (15 seats)		Shared taxi Operation (4 seats)	
	Operator's Costs	Resource Costs	Operator's Costs	Resource Costs
1	159	171	137	133
2	255	282	213	205
3	352	392	286	277

14.4 The costs of taxi operation

The vehicle independent standing costs of operating a taxi service (the control costs) were taken to be the same as the dial-a-bus control costs. The vehicle standing costs and running costs are shown in Table 6 below, where the operator's costs are those of a private operator.

APPENDIX 5 TABLE 6

Taxi operating costs

	Costs	
	Operator's Costs	Resource Costs
VEHICLE STANDING COSTS (£/week)		
Licence	0.23	–
Rent and Rates (garage)	1.73	1.73
Insurance	0.96	0.89
Interest and depreciation	7.82	7.28
Vehicle radio	1.49	1.35
Vehicle radio maintenance	0.55	0.55
Driver's wages	41.86	41.86
Total:	54.64	53.66
RUNNING COSTS (pence per vehicle mile)		
Fuel	1.91	0.97
Lubricants	0.09	0.08
Tyres	0.25	0.23
Maintenance	1.11	1.11
Track cost	–	0.65
Total:	3.36	3.04
SUPPORTING DATA	£	
Cost of fuel (pence per gallon)	53.5	
Fuel consumption (m.p.g.)	28	
Cost of tyres	50	
Mileage life of tyres	20,000	
Cost of vehicle (£)	2,300	
Life of vehicle (years)	8	

Reference 8 indicates that the revenue mileage of a taxi is, on average, equal to 62 per cent of the vehicle mileage. The average number of passengers travelling in a group is assumed to be 1.2, the same as for a dial-a-bus service. On combining this information with the average journey length of 1.1 miles, the vehicle mileage was shown to be related to the demand by the expression

$$\text{Vehicle mileage} = \frac{\text{Average journey length x Demand}}{0.62 \text{ x group size}}$$

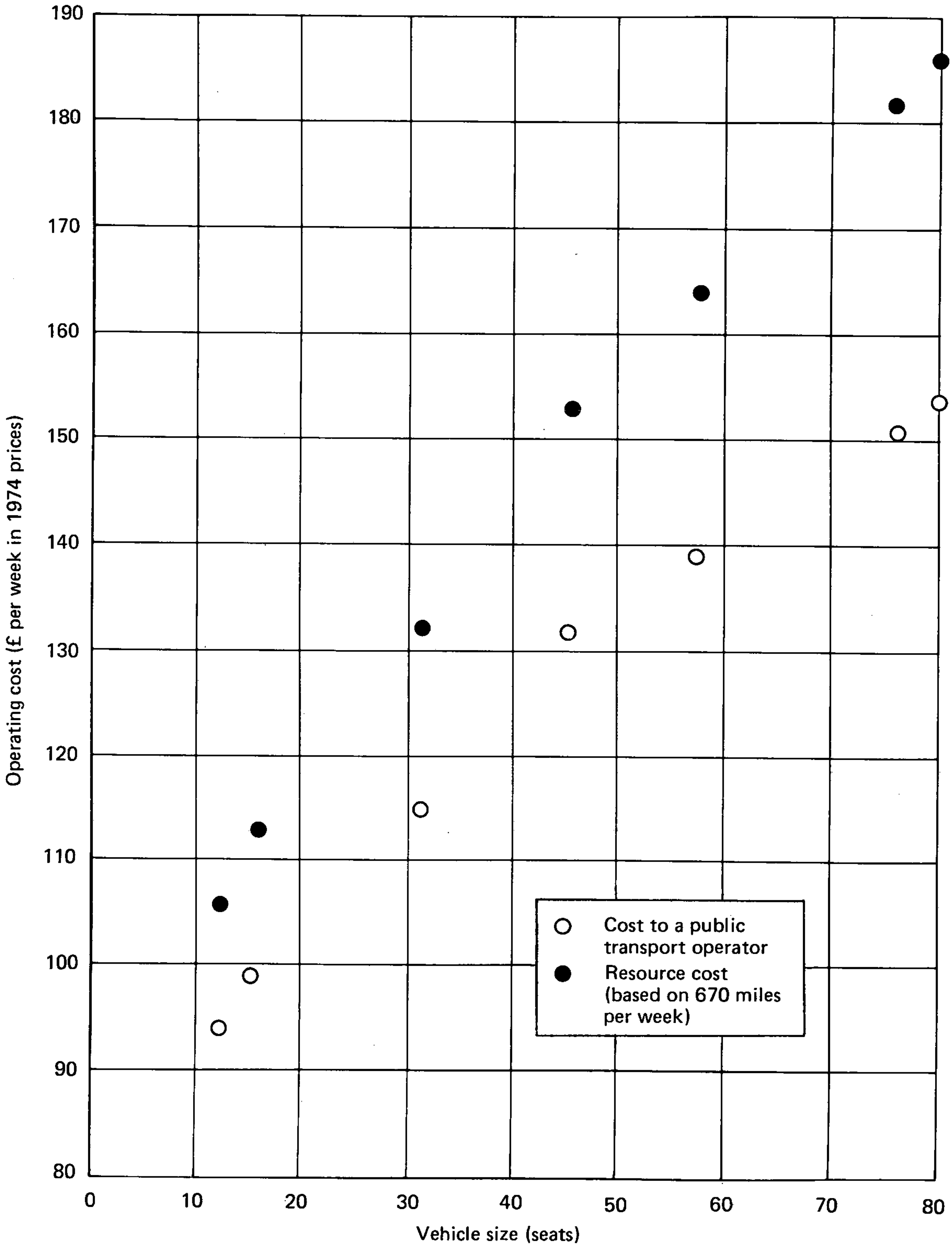
$$= 1.48 \text{ Demand}$$

Combining this expression with the costs shown in Tables 4 and 6 of this Appendix, the costs of operating a taxi fleet are given

$$C = 62 + 55n + 0.050N$$
$$R = 61 + 54n + 0.045N$$

Where C and R are the costs to a private operator and the resource costs, respectively, in £ per week; n is the number of vehicles in the fleet and N is the total demand measured in trips per week.

It may be noted that, as would be expected, a taxi service is less efficient in vehicle miles than an equivalent dial-a-bus service carrying the same number of trips.



Appendix 5 Fig. 1 THE VARIATION OF BUS OPERATING COSTS WITH VEHICLE SIZE

15. APPENDIX 6

THE CONSUMER SURPLUS

The consumer surplus associated with a particular service is a measure of how much more the passengers would be willing to pay to retain that service. As shown in Appendix 6 of Reference 2, this is given by the integral of the demand with respect to fare:

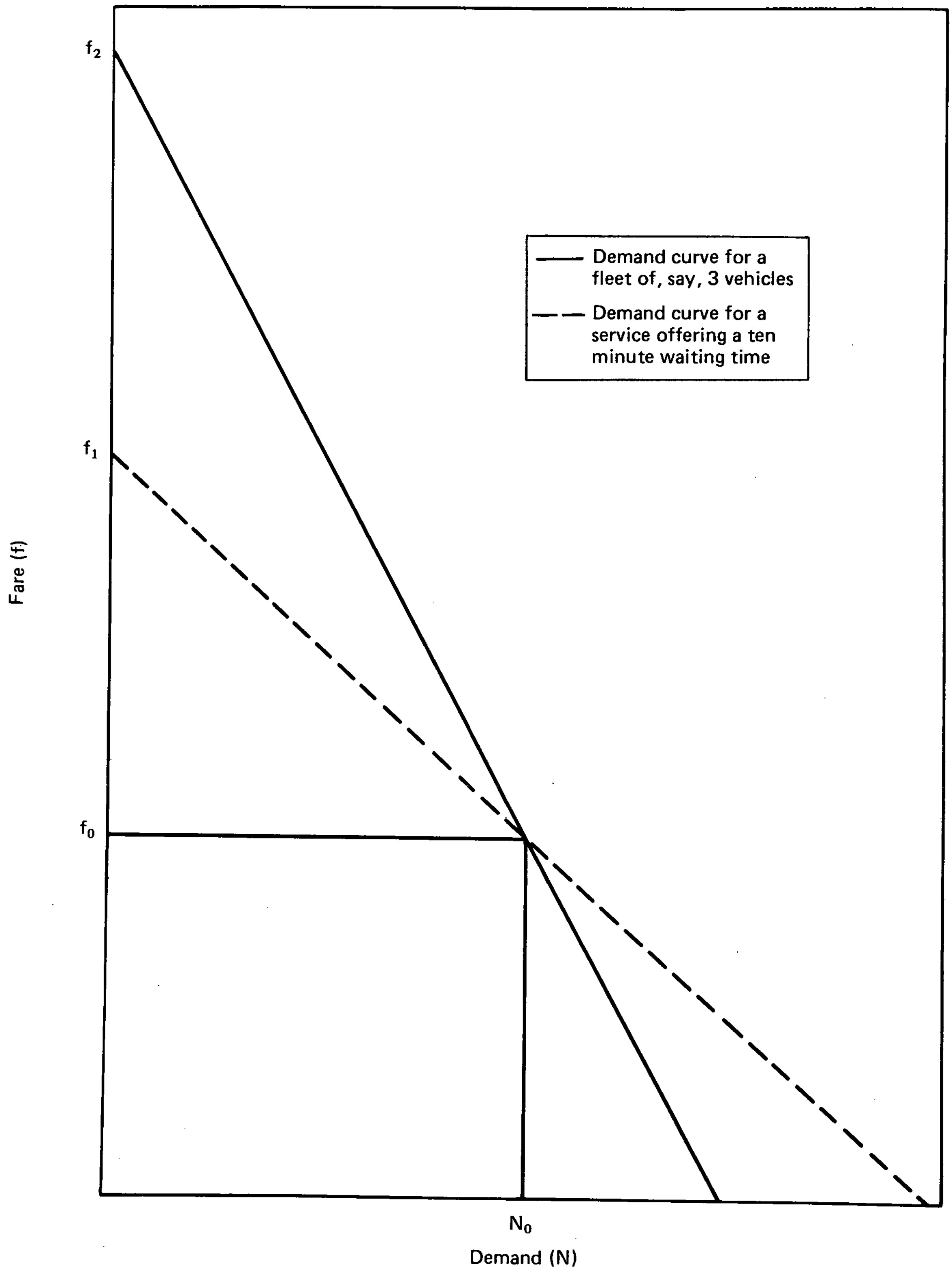
$$CS(f, t) = \int_f^\infty N(f, t)\, df,$$

where t denotes the service characteristics of waiting time, riding time, access and egress time, which must be held constant. This is not the same as holding constant the system characteristics of fleet and vehicle size for, because of the relationship between supply and demand, the service characteristics for a specified transport system will in fact change with the fare. For example, the waiting time for a taxi will reduce as the fare is increased and the demand falls. Thus the service characteristics, t, are a function of the system characteristics, s, and the demand, N:

$$t = t(N, s),$$

and if t is to be held constant in the consumer surplus integral the supply must be adjusted to compensate for any change in demand. Thus the consumer surplus is not the integral under an observed demand curve but that under a demand curve for which the supply is adjusted to any fare change in such a way as to keep the service characteristics constant.

This effect is shown schematically for a taxi service in Figure 1 of this appendix, where for clarity the demand curves are shown as straight lines. In this example, we wish to determine the consumer surplus at fare f_o of a service being provided by three vehicles. At this fare the service carries N_o passengers with a mean waiting time of ten minutes. The two demand curves show the effect of a fare change on the demand (a) holding the fleet size constant and (b) holding the waiting time constant. The first of these is the observed demand curve for which an increase in fare is to some extent off-set by a reduction in passenger waiting time. The demand therefore falls less rapidly than for the case where the waiting time is held constant. The area under the observed demand curve, $\frac{1}{2}N_o\,(f_2-f_o)$, is therefore larger than the consumer surplus which is equal to $\frac{1}{2}N_o\,(f_1-f_o)$.



Appendix 6 Fig. 1 DEMAND CURVES FOR A TAXI SERVICE

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ABSTRACT

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ISSN 0305–1293

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